Health-aware advanced control applied to a gas-lifted oil well network

José Matias * Joachim Ågotnes * Johannes Jäschke *

* Department of Chemical Engineering, Norwegian University of Science and Technology (NTNU), Trondheim, Norway (e-mail: johannes.jaschke@ntnu.no)

Abstract: Currently, preventive maintenance is the common practice in offshore oil production processes. Maintenance stops are scheduled in regular intervals, even if the equipment is working well, sometimes decreasing significantly the plant profitability. In smart production, the goal is to change to a predictive maintenance paradigm, where process monitoring algorithms use process data to detect the system degradation state and predict when the equipment needs maintenance. Adopting this perspective, we propose a model predictive control approach that incorporates process monitoring. This allows us to steer plant degradation actively, preventing violation of health-critical constraints, while optimizing the economic production of the system. In this paper, we present a case study, a gas lift oil well network subject to sand erosion in the choke valves. The results indicate that we are able to maximize oil production while keeping the erosion of the choke, the critical process equipment, below a safe threshold. Additionally, we discuss some of the implementation issues of the proposed approach.

Keywords: Maintenance, Process Control, Optimization, Subsea processes.

1. INTRODUCTION

There is an intuitive trade-off between maximizing production and minimizing equipment degradation. In oil wells, for example, we would like to extract as much oil as possible. However, such strategy has a negative effect on the remaining useful life (RUL) of the equipment. The valves that control the reservoir outflow tend to degrade faster if we increase the throughput, for instance.

Estimating the RUL can be challenging. This topic has received much attention in the last years mostly due to the advent of smart production (He and Wang, 2018). Advances on information, communication, and sensing allow engineers to collect gigantic amounts of process data, which can be used for developing accurate models for estimating RUL. However, the prognostics analysis is typically carried out offline (Si et al., 2012), i.e. the RUL model parameters are estimated using past degradation data of similar systems and they are not adapted as new measurements come.

Since the need for just-in-time maintenance is increasing, new real-time health condition monitoring strategies have been developed (e.g. Hu and Fan (2017) and He and Wang (2018)). They use the current process measurements to adapt the parameters of the RUL estimation model. Consequently, it is possible to draw conclusions in real-time about the system health state. However, the estimated RUL is generally used for making maintenance decisions and is usually not used for continuously adapting the production rate, or other operational parameters.

We propose to integrate both in a process control framework. Our strategy aims at optimizing production while proactively controlling system degradation rather than simply reacting to it. Balancing both contradicting objectives, we can achieve not only a cost-effective operation but also a safer one. Potentially, it could prevent damage to valuable equipment, human life losses and even environmental disaster due to system overuse. In addition, we can ensure that the facilities remain operational until the next planned maintenance, with no unplanned shutdowns due to premature equipment failures. Our framework, which combines dynamic optimization strategies with degradation prognostics and system health estimation, is based on Verheyleweghen and Jäschke (2017). Similar frameworks have been investigated by a few authors in recent years (see, Pereira et al. (2010); Escobet et al. (2012)).

In Verheyleweghen and Jäschke (2017), the authors proposed a real-time optimization (RTO) strategy that combines the process model with equipment degradation models. The optimization problem is solved repeatedly in a shrinking horizon fashion, until the next planned maintenance stop. In our approach, we pose the problem as a receding horizon control problem instead (Rawlings et al., 2017). The idea is to find the optimal input trajectory given the current plant condition, then implement only the first input of the sequence before the model is re-optimized in the next sampling time. By considering shorter time horizons, we have faster responses regarding the system degradation. Moreover, since we re-optimize the system more frequently, we can rely on more frequent RUL estimates feedback to deal with uncertainty, instead of using a scenario based approach like in Verheyleweghen and Jäschke (2017). Our strategy is tested in a case study, a

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gas lift oil well network. The results show that, we not only enhance operational performance but also prevent violation of health-critical constraints.

The remainder of the paper is structured as follows: In Section 2, we briefly introduce our strategy. Next, Section 3 presents the optimal control problem (OCP) and, then, we show how to fit our strategy into an OCP framework. Section 4 shows the process description of the gas lift well network. We present the results of the case study in Section 6. Finally, in Section 7, the implementation challenges of our strategy are discussed.

2. GENERAL DESCRIPTION

In order to implement our controller that optimizes the economics while taking into account the current and future degradation, we have the following step-wise procedure, which is also summarized in the diagram of Figure 1.

- (1) Get measurements y from the plant, which may include measured equipment health indicators;
- (2) Estimate the current states \hat{x} and system health $\hat{\epsilon}$;
- (3) Use the estimates to compute the optimal operation strategy u^{*} taking into account the RUL estimating model. The goal is to find the sequence of inputs u's that minimizes costs (or maximizes profit) while avoiding violation of safety constraints. Operational constraints, like maximum processing capacity, are also considered.

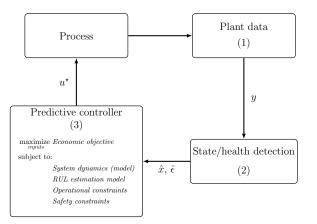


Fig. 1. Block diagram of the proposed control strategy that combines advanced optimizing control with process monitoring for predictive maintenance.

3. CONTROL STRATEGY

We next introduce the optimal control problem. Then, we show how to pose our problem using the OCP framework.

3.1 Predictive Optimizing Control

The OCP is usually solved to determine an optimized future trajectory for time-varying systems, in which an objective function is minimized/maximized subjected to a set of constraints. The OCP solution is usually obtained by integrating the system dynamic model at discrete time intervals (Biegler, 2010). Often, when the OCP is used

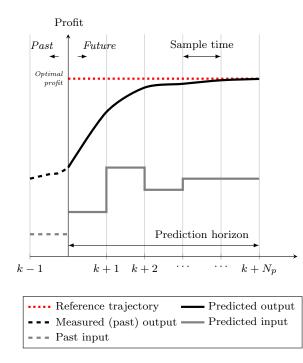


Fig. 2. Description of an economic model predictive controller working

for solving an economic-based problem, it is referred to as Economic Model Predictive Control (EMPC) or Dynamic Real-time Optimization (DRTO) (Rawlings et al., 2017). Figure 2 shows a schematic of an EMPC, in which the objective is to maximize profit.

At each time step k, the EMPC predicts the optimal trajectory for the inputs (gray line) based on the current state of the system and a reference trajectory (red dotted line). Instead of predicting the trajectory to the infinity, which is unfeasible, the prediction horizon is truncated to $k + N_p$. If the system can be controlled, we can find a set of input moves $u^* = [u_k^*, u_{k+1}^*, \ldots, u_{k+p}^*]$ that optimize process operation in the prediction horizon.

Instead of applying the whole sequence of inputs u^* , the controller implements only the first control move u_k^* and then, after one sampling time, the OCP is solved again using the new information at k + 1. The process is solved in this receding horizon fashion because the reference trajectory may change, or updated measurements may have updated the model parameter or state estimates Rawlings et al. (2017).

The objective function is determined in order to reflect the control/optimization goals. There exists several types of objectives for the MPC, such as regulatory (where the controller tries to minimize the difference between the predicted output and the reference trajectory) or purely economic (e.g. maximizing profits or reducing costs, like shown in Figure 2).

3.2 OCP problem for our strategy

Since our goal is to maximize profit, we formulate the objective function accordingly. The OCP problem can be written as:

$$\underset{u}{\text{maximize}} \int_{t_0}^{t_f} (profit(t) - 0.5 \ \dot{u}(t)^T \tilde{Q}_u \dot{u}(t)) \ dt$$

subject to: (1)

$$\begin{split} \dot{x} &= \hat{f}(x, u) \\ \dot{\epsilon} &= \tilde{g}(x, u) \\ 0 &\leq \tilde{h}(x, u, \epsilon) \end{split}$$

where, x are the states of the system, which may have physical meaning (e.g. pressures, flowrates, etc.) or not; uare the system inputs, usually valve openings; ϵ represents the degradation of the components of interest; The dot notation, e.g. \dot{x} , indicates the time derivative of the variable; $\tilde{f}(\cdot)$ is the system model, $\tilde{g}(\cdot)$ is the model for estimating RUL; and $\tilde{h}(\cdot)$ are the operational constraints, like maximum capacity of a compressor. In \tilde{h} we also include the health-critical constraints that need to be enforced. Crack length in the compressor bearings, for instance.

The objective function is the integration, from t_0 to the end of the prediction horizon t_f , of the operational profit plus a regularization term on the input change, which is defined as $\dot{u}(t)$. This term aims at minimizing the change of inputs during one sample time. The weight \tilde{Q}_u and the prediction horizon t_f are tuning parameters.

To solve the dynamic problem of Equation (1), we discretize the system using orthogonal collocation with three collocation points (Biegler, 2010). The problem becomes:

$$\begin{array}{l}
\text{maximize} \sum_{k=t_0}^{t_0+Np} \left(profit_k - 0.5\Delta u_k^T Q_u \Delta u_k \right) \\
\text{subject to:} \\
x_{k+1} = f(x_k, u_k) \quad \forall k = t_0, \dots, t_0 + Np \\
\epsilon_{k+1} = g(x_k, u_k, \epsilon_k) \quad \forall k = t_0, \dots, t_0 + Np \\
0 \le h(x_k, u_k, \epsilon_k) \quad \forall k = t_0, \dots, t_0 + Np
\end{array} \tag{2}$$

in which, k represents the k^{th} sampling time and N_p is the prediction horizon in discrete form (i.e. number of sampling times to the end of the horizon). f, g and h result from the discretization of the continuous-time models \tilde{f}, \tilde{g} and \tilde{h} . The input change is also defined in discrete terms as $\Delta u_k = u_k - u_{k-1}$. Q_u is the discrete counterpart of \tilde{Q}_u .

3.3 Including degradation in the control formulation

One implementation issue of this strategy arises when the degrading state is represented by an integrating process. This type of process is characterized by the fact that the open-loop system response does not stabilize at a given value if a step change in the input is applied, meaning that the process does not have the same output given the same set of inputs and disturbances.

The level of liquid in a drum is an example of an integrating process. If one changes the inlet flowrate while the outlet flowrate remains constant, the level increases indefinitely. For degradation, the behavior can be similar. Let us say that degradation mechanism of a given equipment is related to fluid flow. Therefore, even very small flowrate values result in equipment wear. Since we consider long prediction horizons for the controller, it may be unable to keep the degradation below the maximum threshold during the whole prediction interval, if we already have a significant level of equipment degradation initially. As a consequence, the control problem will fail to converge and to provide an input value to the process, which may lead to infeasibility.

A contribution of this paper is to devise a strategy to cope with this problem by including a slack variable s in the maximum allowable degradation constraint $(0 \le \epsilon + s \le \epsilon_{max})$ and in the objective function:

$$\sum_{k=t_0}^{t_0+N_P} (profit_k - 0.5\Delta u_k^T Q_u \Delta u_k + Q_s s_k)$$
(3)

This slack term gives the controller the opportunity to violate the constraint, but at a high cost for the objective function as the weight Q_s is large. The slack variable ensures that the optimization problem of Equation (2) gives a feasible solution even if the degradation level at the beginning of the horizon is high. It is important to keep in mind that the maximum threshold would be violated during the prediction horizon, only if we apply the complete sequence of input moves computed at the current time step. However, we only implement the first input move and then recompute the sequence. If the predictions sequentially indicate that the safety constraint will be violated, a maintenance stop needs to be scheduled or a different operational strategy needs to be devised.

4. CASE STUDY: GAS LIFT OIL WELL NETWORK

We illustrate how our strategy work in a subsea 3-well gas lift network. Subsea wells are usually placed on the seabed, connecting the oil and gas reservoir to topside facilities, for example a fixed platform or a Floating Production Storage and Offloading (FPSO). The reservoir outflow is controlled by choke valves, which are prone to erode with time. Repairing or replacing them is an issue because maintenance interventions on the equipment, which is on the seabed, are very expensive. Unanticipated breakdowns can lead to long halts and large losses in the production.

The erosion is intensified by reservoir sand production, particle erosion can severely limit the RUL of the valves. Usually, the sand managing strategy is outlined early in the field development to ensure appropriate sizing and selection of equipment as well as instrumentation for monitoring, controlling and handling of sand production. Even with all these precautions, a very conservative operational strategy is often adopted. Typically, an acceptable sand rate is defined based on worst-case erosion scenarios, leading to sub-optimal operation and potential profit loss (Verheyleweghen and Jäschke, 2018).

This conservativeness can be reduced by applying our control strategy to this process. Then, we can achieve an optimal operation and prevent violation of healthcritical constraints. In order to show how to implement our strategy in the gas lift oil well network, we describe the system model, then, discuss how we estimated the RUL of the choke valves. Since the erosion mechanism of valves is reasonably understood, we choose to use simplified semiempirical models for calculating the RUL of the chokes. We want to enforce that the valve erosion remains below

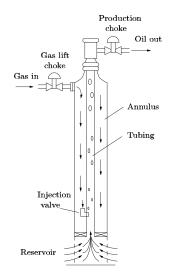


Fig. 3. Simplified representation of a gas lift well (Eikrem et al., 2004). The well is composed by the *annulus*, a void between the product pipeline and the external tubing where the gas lift is injected, and the *tubing*, where the oil and gas mixture flows.

a maximum allowable threshold. Finally, we show how to include both model in the framework of Equation (2).

4.1 Gas lift Model

Gas lift is used in wells when the reservoir pressure is not enough to drive the oil from the reservoir to the processing facilities at the sea level. A simplified representation of the well is shown in Figure 3. By injecting gas into the well through the annulus, the density of the fluid mixture in the tubing decreases resulting in lower hydrostatic pressure losses along the well. Consequently, the reservoir outflow increases. The model used for describing the gas lifted well system is based on Krishnamoorthy et al. (2016). The mass balances in each well are:

$$\dot{m}_{gt} = w_{lg} + w_{rg} - w_{pg}, \quad \dot{m}_{ot} = w_{ro} - w_{po}$$
(4)

where, w_{lg} is the gas lift mass flow rate, w_{rg} and w_{ro} are the gas and oil mass flowrates from the reservoir, and w_{pg} and w_{po} are the mass flowrates of the produced gas and oil. m_{gt} and m_{ot} are the gas and oil mass holdup in the well. Both the gas lift and total production rate $(w_t = w_{pg} + w_{po})$ are adjusted through choke valves:

$$w_t = C_{pc}\sqrt{\rho_w(p_{wh} - p_{out})}$$

$$w_{lg} = C_{iv}\sqrt{\rho_a(p_a - p_{wi})}$$
(5)

where, C_{pc} and C_{iv} are the valve coefficients of the production choke and injection valve. p_{wh} , p_{out} , p_a and p_{wi} are the pressures at the well head, well outlet, annulus and injection point, respectively. ρ_a and ρ_w are the densities in the annulus and in the well tubing, which are given by:

$$\rho_a = \frac{Mp_a}{T_a R}, \quad \rho_w = \frac{m_{gt} + m_{ot} - \rho_o L_r A_r}{L_w A_w} \tag{6}$$

where, M is the gas lift molar mass, R is the universal gas constant, and T_a is the temperature in the annulus. L_r and A_r are the lenght and cross-sectional area of the tubing above the gas injection point, and L_w and A_w below it. ρ_o is the oil density. Note that, for calculating the gas density, we assume ideal gas law and constant temperatures along the wells, as well as constant oil density. The reservoir outflow is computed by:

$$w_{ro} = PI \cdot (p_r - p_{bh}), \quad w_{rg} = GOR \cdot w_{ro} \tag{7}$$

where, PI and GOR are the productivity index and gasoil-ratio, and p_r is the reservoir pressure. They are wellspecific parameters, which need to be adjusted accordingly.

4.2 Erosion Model

Chokes come in many different forms and geometric layouts which makes it difficult to find a generic model. We use the choke erosion model of DNV (2015), which gives the erosion rate with an uncertainty factor of at least 3 times. If a more accurate measurement is needed, Computational Fluid Dynamics (CFD) is usually used in each specific case (DNV, 2015). However, since we need to include this model in the controller and solve it in a reasonable time frame, we can use the simpler, yet inaccurate, model and rely on the feedback to correct possible modeling errors. The erosion rate is given by the semiempirical relationship:

$$\dot{\epsilon} = \frac{K \cdot F(\alpha) \cdot U_p^n}{\rho_t \cdot A_t} \cdot G \cdot C_1 \cdot G_f \cdot \dot{m}_p \cdot C_{unit}$$
(8)

where, ϵ is the erosion in mm; K is the material erosion constant; n is the velocity exponent; ρ_t is the valve material density; C_1 and G_F are geometry factors; C_{unit} is a unit conversion factor; \dot{m}_p and U_p are the mass sand rate and the impact velocity, respectively, they are the equation variables. The others are model parameters and can be set once the choke valve is specified. Here, the sand rate is assumed to be constant and known.

4.3 Control strategy

The system model equations and the erosion model are used as constraints to the OCP problem. However, since the time scale of the pressure and flow changes is smaller when compared to the time scale of the erosion, the flow and pressure dynamics are neglected (i.e. \dot{x} is set to 0). Such assumption leads to a Differential Algebraic Equation (DAE) system, which is a combination of differential equations and algebraic equations. The resulting gas lift model is:

The manipulated variables/inputs u are the gas lift flow rates of the wells and the states x are the pressures, flow rates and mass holdup along the wells. In the operational constraints h, we included constraints on the inputs $(u_{min} \leq u \leq u_{max})$, on the input variation Δu $(-\Delta u_{max} \leq \Delta u \leq \Delta u_{max})$, and on the erosion ϵ $(0 \leq \epsilon + s \leq \epsilon_{max})$:

$$0 \le h(x, u, \epsilon) \quad \begin{cases} input bounds\\ max. input variation\\ max. allowable erosion \end{cases}$$
(10)

The variable s is the slack term, which is also added to the objective function:

$$\sum_{k=t_0}^{t_0+Np} \left(\sum_{i=1}^3 w_{t,i,k} - 0.5\Delta u_{i,k}^T Q_u \Delta u_{i,k} - Q_s s_{i,k} \right) \quad (11)$$

According Equation (8), we note that the erosion is an integrating process. The time variation of the erosion is linearly proportional to the particle impact velocity U_p , which in turn is a function of the gas lift flowrate. When the controller realizes it may violate the erosion bounds, it will try to change the gas lift rate during the prediction horizon in a way that does not violate this condition. However, it may be impossible to do so, given that there will be erosion even at the lowest value of the gas lift flow rate, and it may enter an infeasible region where the controller cannot satisfy the constraint on the maximum erosion. The slack term, then, allows this constraint to be violated but with a penalty to the objective function.

5. SIMULATION

In the simulations, we considered that the actual process is represented by the same model used in the controller. However, instead of measuring all the states x, the measurements y contain only the pressures and the oil and gas production flow rates (at topside). Additionally, we assume that the erosion measurements are obtained via a soft sensor.

The measurements, including the erosion, contain noise in the range of 1% (i.e. $y_{noise} = y_{model}(1+0.01r)$, where r is drawn from a standard normal distribution). The noise is uncorrelated in time and between measurements.

The tuning values are obtained by trial and error for this process, they are shown in Table 1. Note that the erosion is a very slow process. Hence, a long prediction horizon is needed for the controller to be able to predict the erosion effectively.

Table 1. Simulation parameters.

Parameter	Value
ϵ_{max}	$2.1 \; [mm]$
N_p	$100 [\mathrm{day}]$
Q_u	$\operatorname{diag}(1\ 1\ 1)$
Q_s	99999
u_{min}	$0.4 \; [kg/s]$
u_{max}	2 [kg/s]
Δu_{max}	$0.01 \; [kg/s]$

The goal is to maximize the objective function of Equation (3), while keeping the erosion of each production choke below ϵ_{max} during the interval of 500 days. All of the system simulations are done in MATLAB. For integrating the system and for optimization purposes, CasADi is used (Andersson et al., 2019). CasADi provides useful tools for nonlinear optimization and is an open-license software. The IPOPT solver, embedded in CasADi, is used to solve the non linear programming problems. Orthogonal collocation is applied for solving the differential algebraic equation of Equation (9). The algebraic equations are implemented as constraints at every collocation point.

6. RESULTS

The results show how the MPC controls the choke erosion in the 3-well gas lift network. After simulating the MPC for 500 days, the results are shown in Figure 4.

Initially, the controller maximizes the total production of oil by increasing the gas lift rate in all the wells as fast as possible, only being held back by the limit of Δu_{max} . Since the gas lift rate is at its maximum limit and constant, the slope of the erosion and the total production of oil is also constant.

Due to the reservoir characteristics, well 1 is the least exposed to erosion. Therefore, the controller tries to extract as much oil as possible from this well. On the other hand, the second well has the highest extent of erosion when applying the same gas lift flow rate. Around 400 days, the controller predicts that, if the gas lift flowrate remains at the same level, the constraint on the erosion threshold of well 2 will be violated. Then, it starts decreasing the gas lift rate of the second well. Next, around 450 days, the controller takes the same action regarding well 3, decreasing its production gradually.

The total production of oil is the highest when the gas lift rate is at the maximum for all the wells. When the controller detects that the erosion in well 2 and after in well 3 will eventually reach the erosion threshold, the MPC reduces the production in the specific well, decreasing the gas lift rate. Consequently, the total production is reduced. The controller does not violate the constraint on erosion and is thus successful in keeping the erosion in each well below the set threshold while maximizing the production, which was our initial goal.

7. DISCUSSION

Despite the fact that this is an ideal set-up for our problem (with a simple erosion model, a relatively accurate healthstate estimation of the valve, and a known sand rate), we use the simulation results to illustrate how the method behaves and justify further investigation on the results. The aim of the case study is to show case our approach. According to the simulation results, we conclude that we can increase the production while making sure that the critical levels of erosion are not exceeded. Only by including a model for estimating the RUL in the classical MPC cycle. For more robust results, a Monte Carlo study should be performed in order to average out particular noise realizations.

Although there is considerable active research on the field, obtaining a reliable degradation model for the equipment is still difficult in real applications. RUL estimating models are usually based on degradation data and/or phenomenological knowledge about the process (Shahraki et al., 2017). The first category relies on statistical or artificial intelligence models and is specially useful when the underlying degradation mechanism is poorly known. On the other hand, if the equipment failure mechanism is understood, physic-based models can be used for computing the RUL. For a detailed review of both categories, see Shahraki et al. (2017).

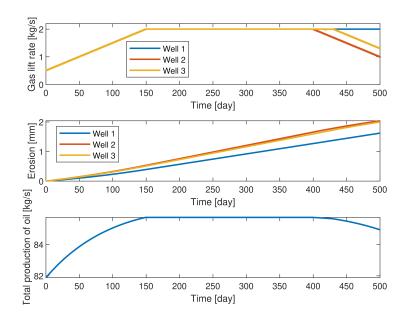


Fig. 4. Results from the MPC simulation for 500 days. The figure on top shows the system manipulated variables, the gas lift flow rate of each well. The figure on the bottom, the total oil production of the well network. The current valve erosion value ϵ and the maximum erosion threshold, which is the same for all the wells, is shown in the middle.

8. CONCLUSION

We presented a framework that combines health detection, prognostics and control. By including health indicators in an MPC framework, our approach can lead to a more profitable and safe operation. Additionally, assets that are properly managed can survive for longer periods, optimizing life cycle and reducing environmental impact.

An accurate model for estimating RUL is a key requirement for the success of our strategy. Normally, this model is built based on field data or on data provided by the equipment vendors. However, obtaining reliable models for estimating RUL can be challenging even with the appropriate data set at hand (Shahraki et al., 2017). In future work, we intend to implement the MPC in an experimental rig for validating our approach.

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