Combined production and maintenance optimization

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SUBSEA PRODUCTION AND PROCESSING

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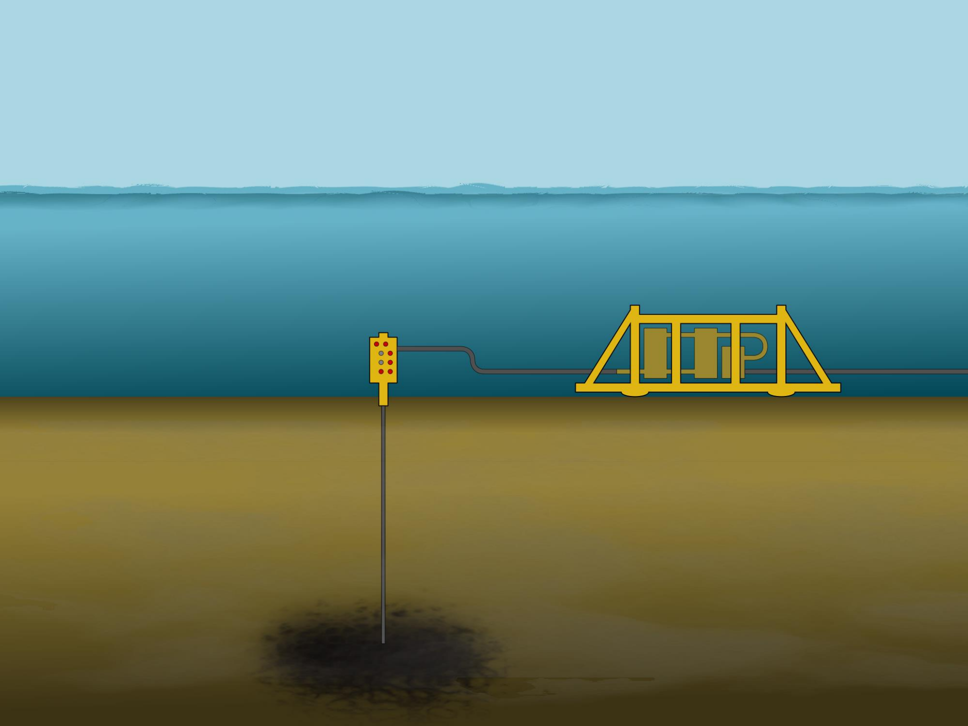
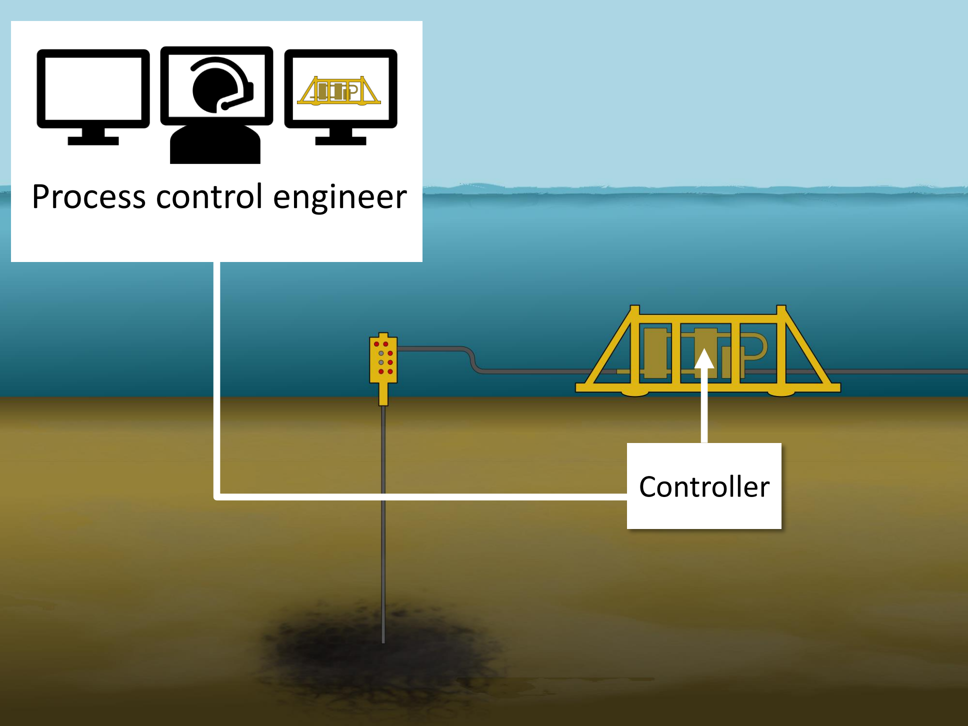
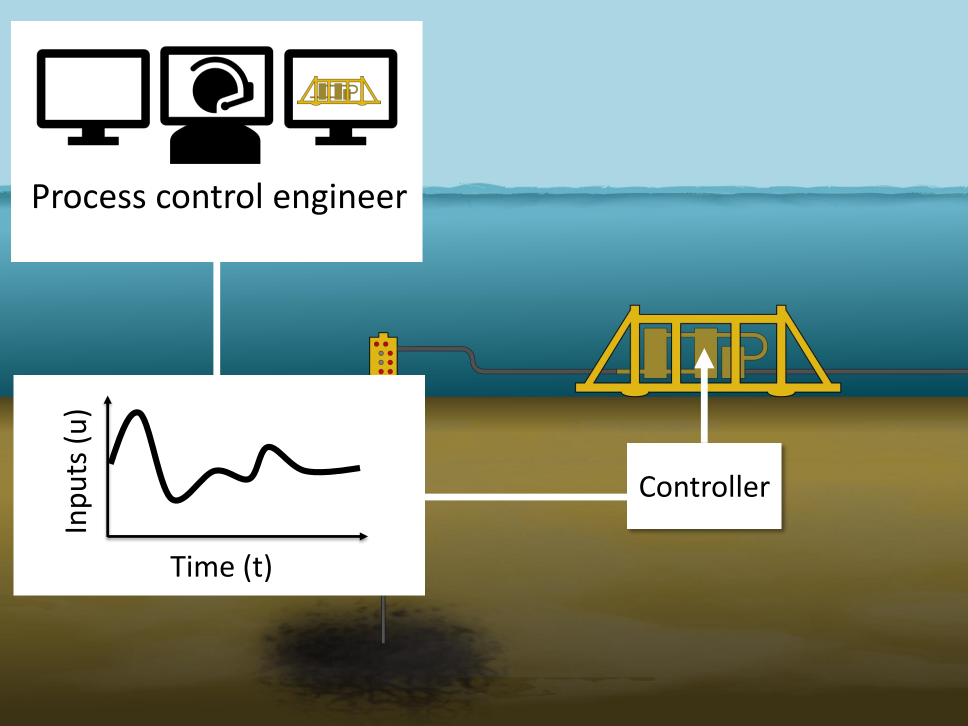
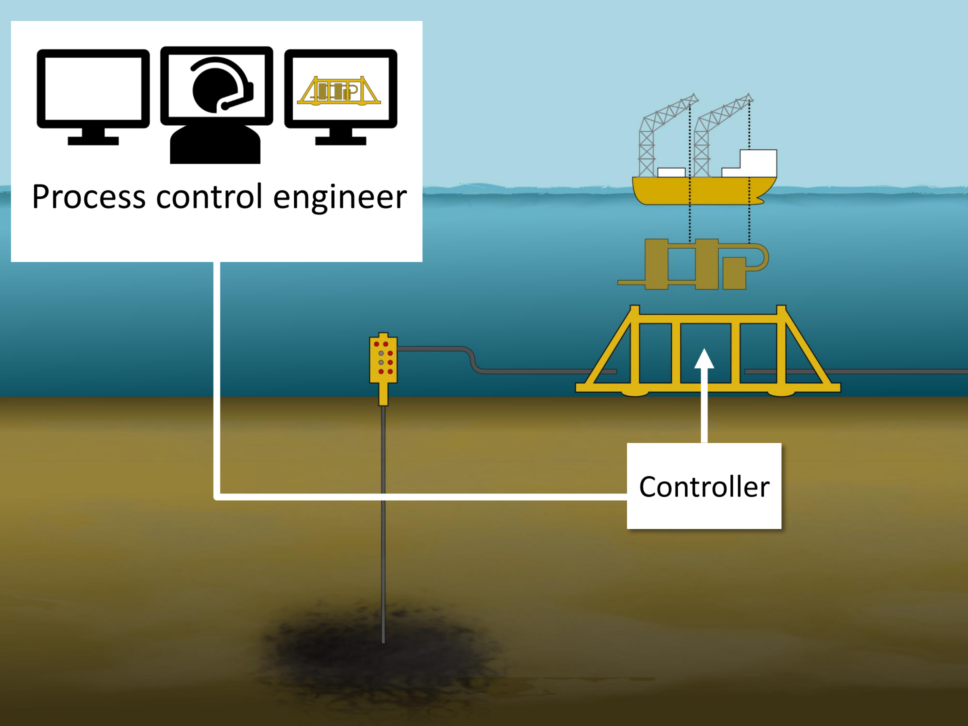



Process control engineer
Controller


Process control engineer
Inputs (u)
Time (t)
Controller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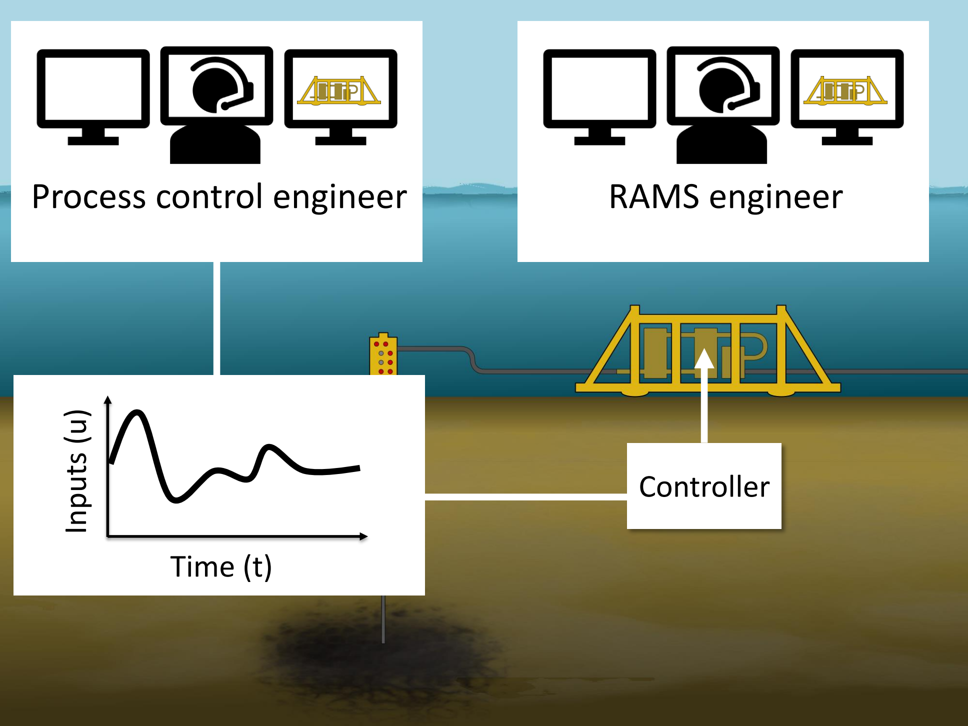
Controller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RAMS engineer
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Time (t)
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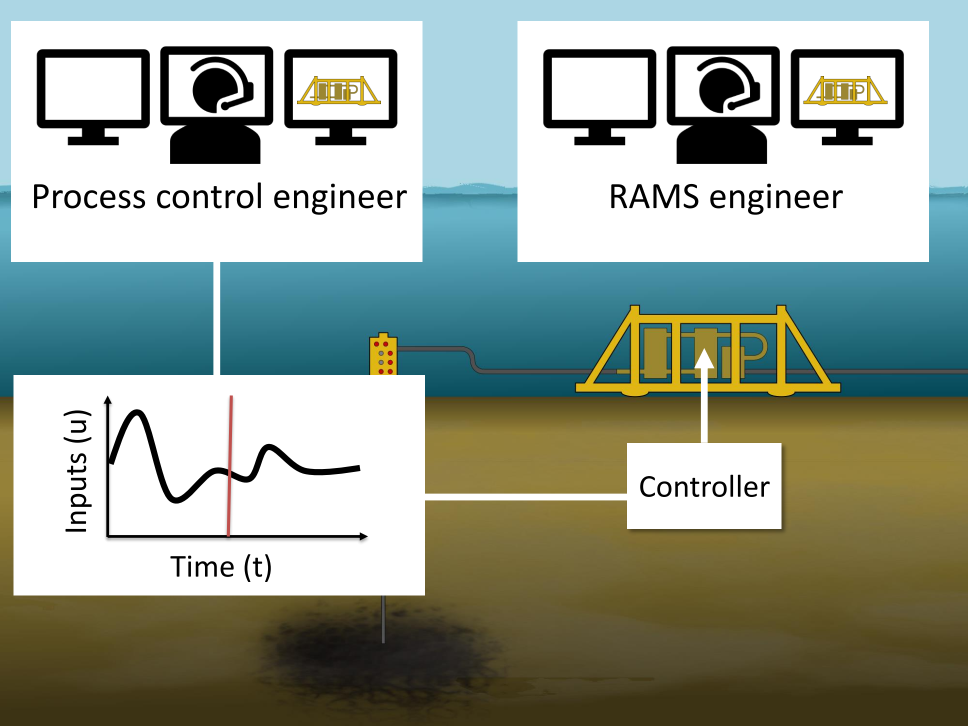
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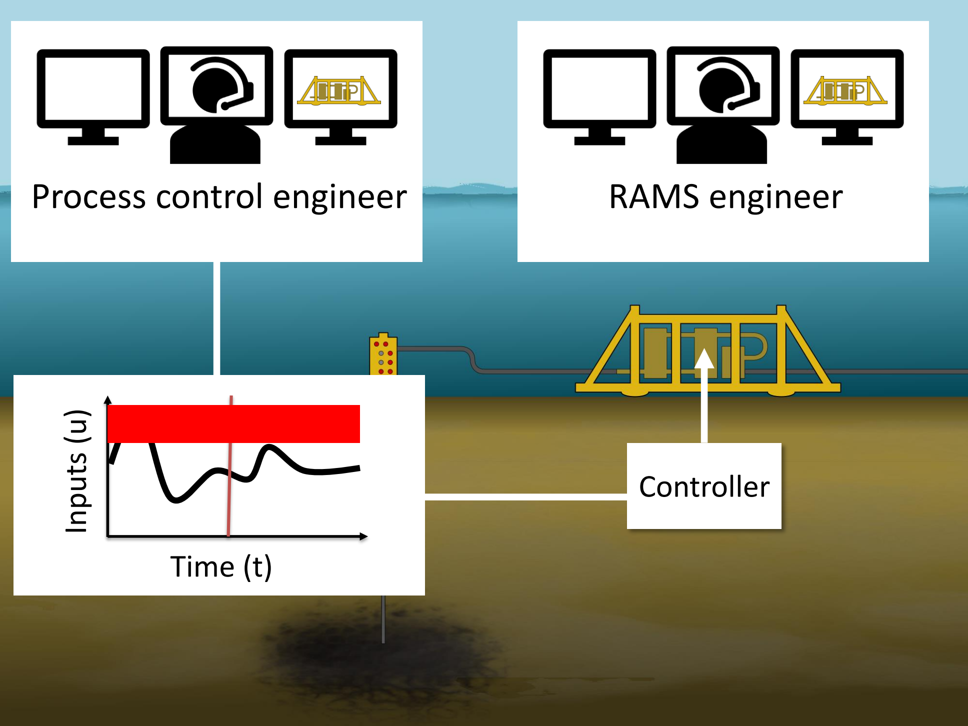
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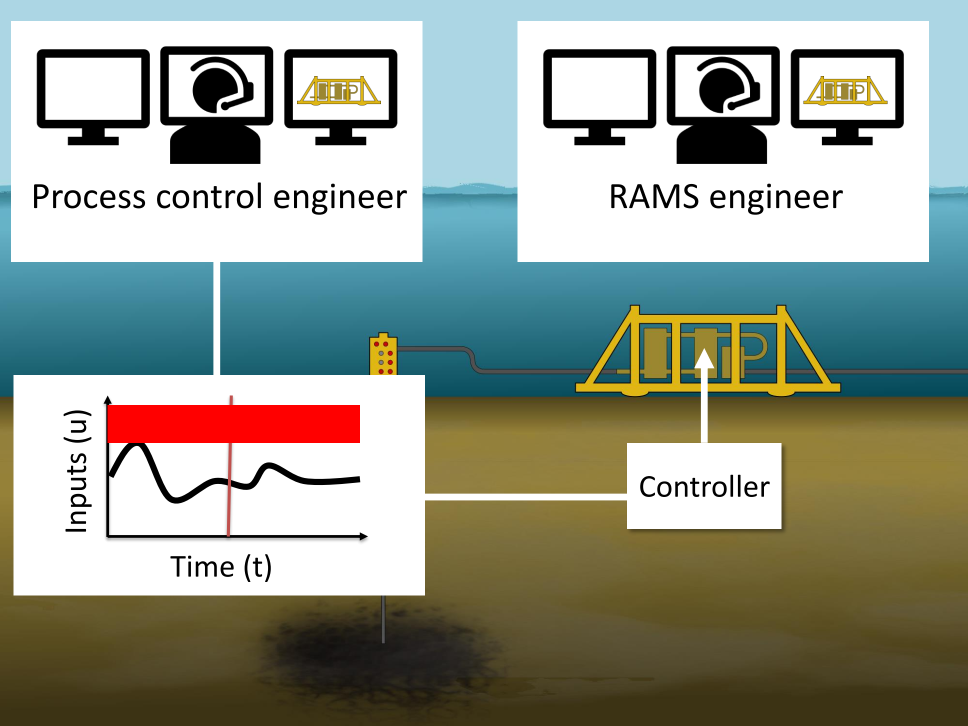
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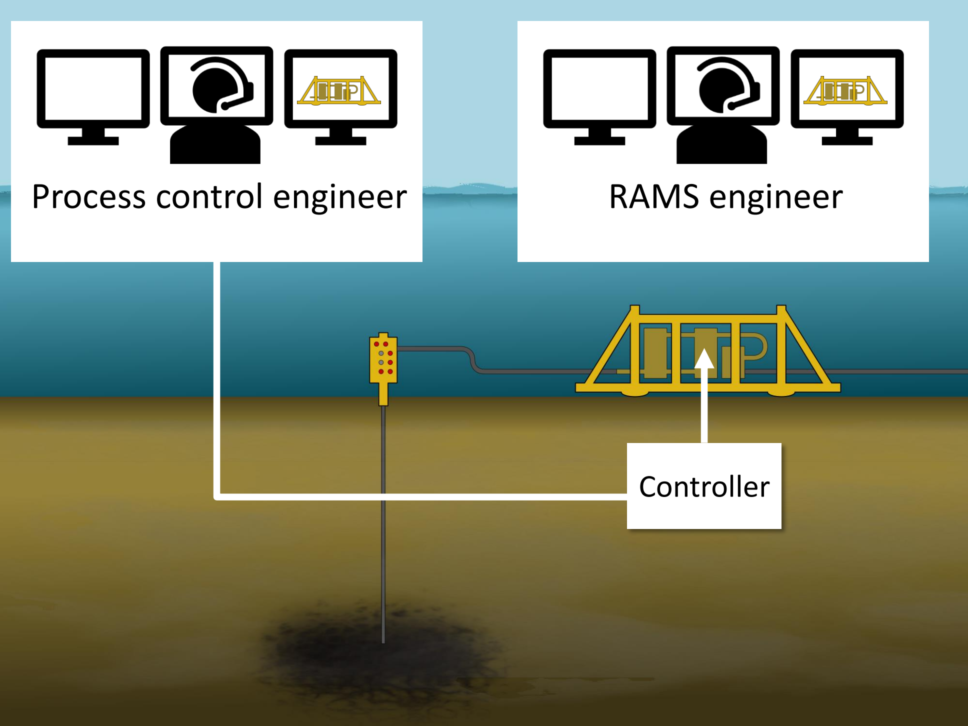
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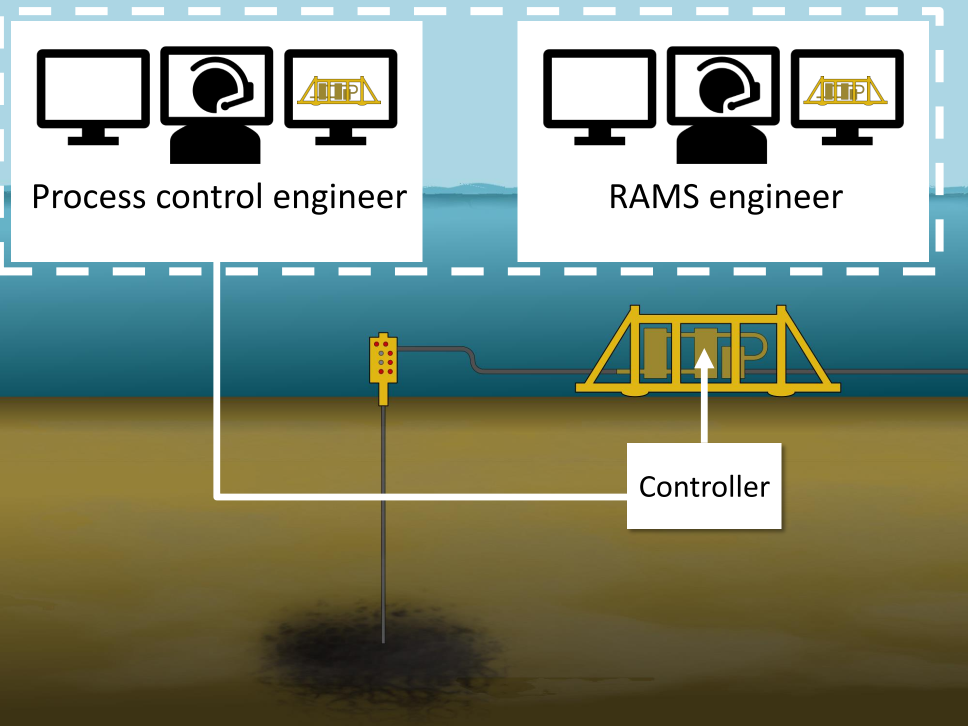
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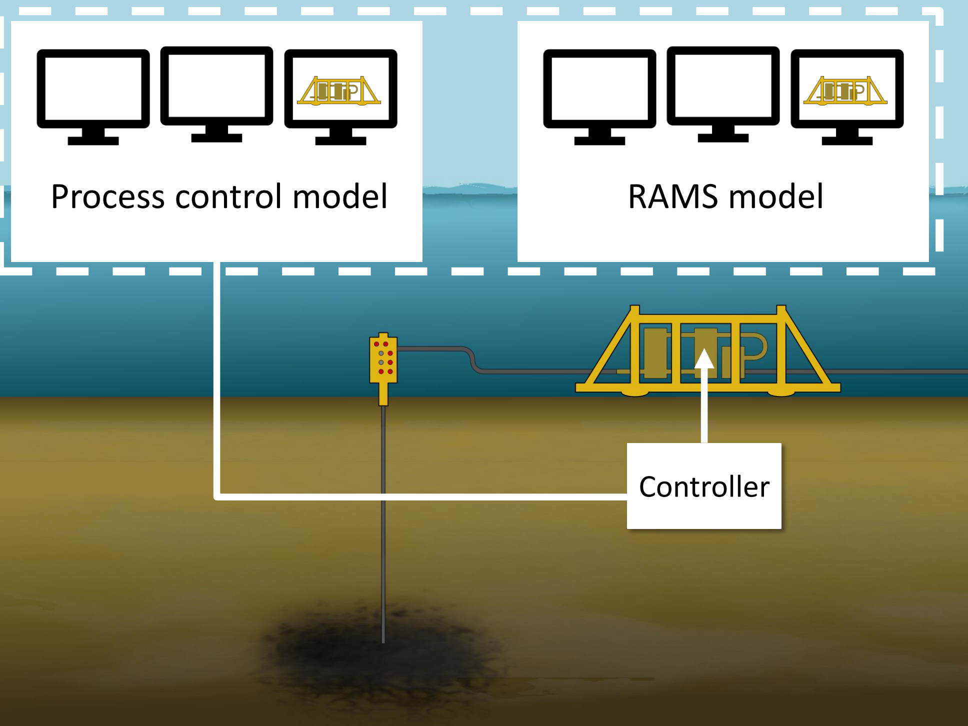
Process control model
RAMS model
Controller

Two main questions:

1. How do we operate optimally, knowing that we will perform maintenance in the future?
2. How often should we perform inspections and maintenance?

Combined maintenance and production optimization

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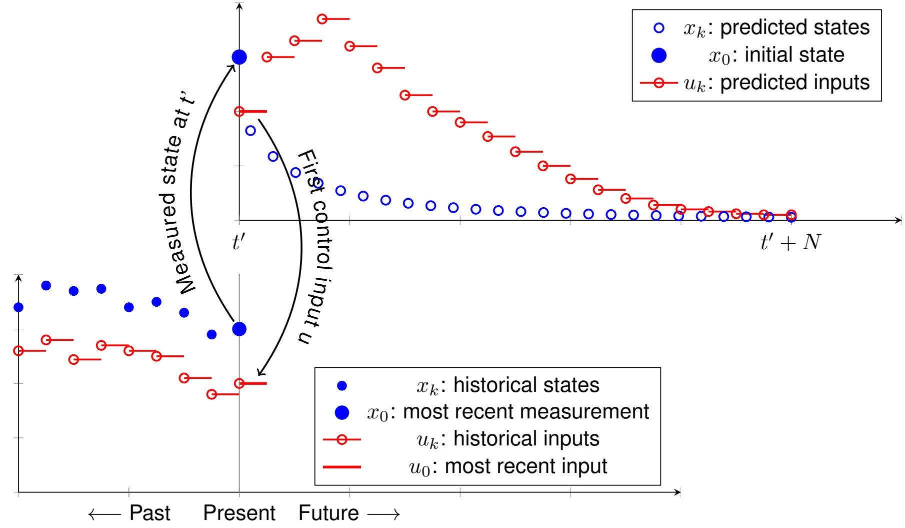
These problems are often solved with Monte Carlo simulations.

3. Can we come up with a more efficient optimization-based approach?

Dynamic optimization

Dynamic optimization



Dynamic optimization

$$\max_{u} \quad \int_0^{t_f} \phi\left(\dot{x}, x, z, u, \pi\right) dt \tag{1a}$$

subject to

$$0 \quad = \quad g(\dot{x}, x, z, u, \pi) \quad \left\{ \begin{array}{l} \text{System dynamics} \\ \text{and model equations} \end{array} \right. \tag{1b}$$

(1c)

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$$0 = h(\dot{x}, x, z, u, \pi) \quad \left\{ \text{System degradation} \right. \tag{1c}$$

Markov chain

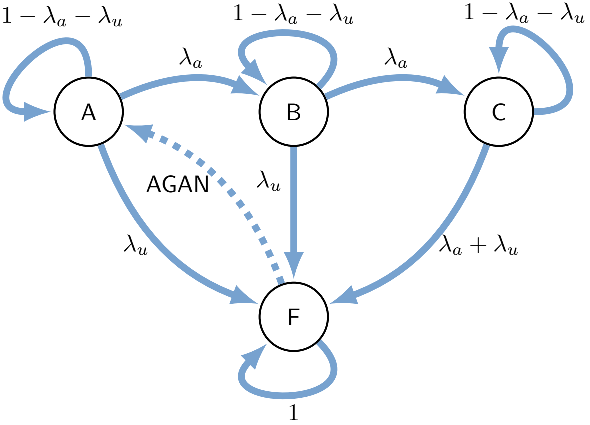

Figure: Markov chain

Markov chain

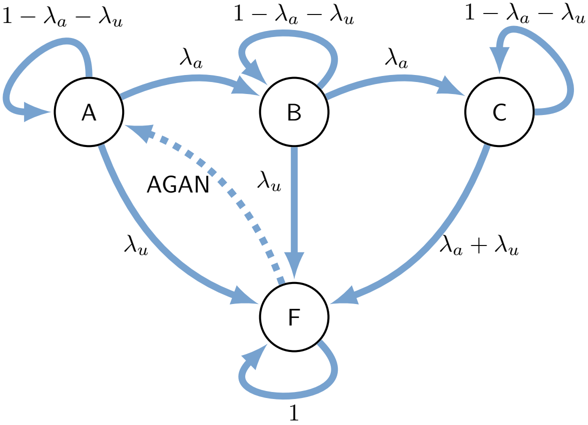

Figure: Markov chain

- Repairs follow As-Good-As-New (AGAN) strategy

Modeling the probabilities

$$\frac{d\boldsymbol{\mu}}{dt} = \underbrace{\begin{bmatrix} -\lambda_u - \lambda_a & 0 & 0 & 0 \\ \lambda_a & -\lambda_u - \lambda_a & 0 & 0 \\ 0 & \lambda_a & -\lambda_u - \lambda_a & 0 \\ \lambda_u & \lambda_u & \lambda_u + \lambda_a & 0 \end{bmatrix}}_{\boldsymbol{\Lambda}} \boldsymbol{\mu} \qquad (2)$$

$\mu_i(t)$: probability of being in state i at time t

Modeling the probabilities

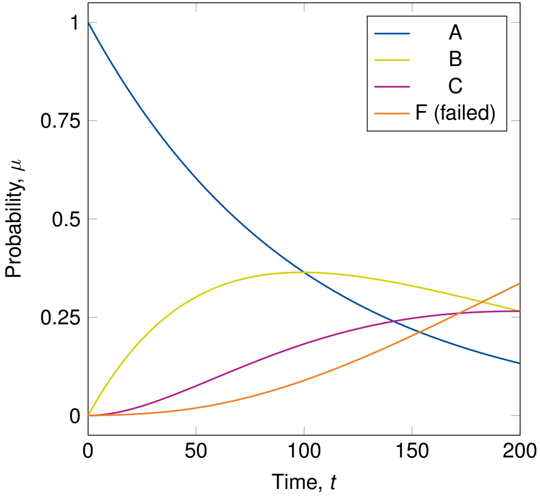

Figure: Evolution of the probabilities without control

Modeling the probabilities

How do the inputs / operating modes enter in the model?

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\Lambda}(u,t) \cdot \boldsymbol{\mu}(t) \tag{3}$$

Modeling the probabilities

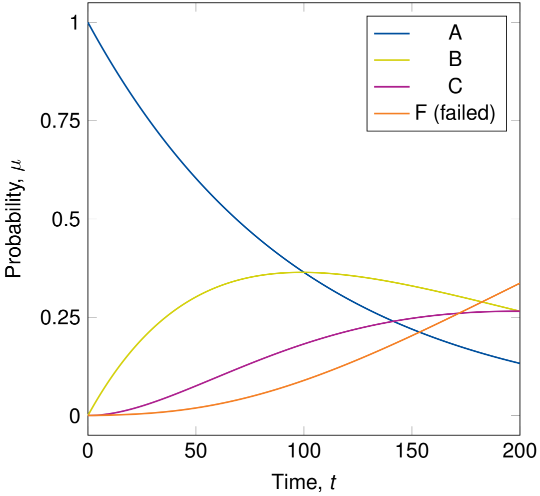

Figure: Solid line: $u = 1$.

Modeling the probabilities

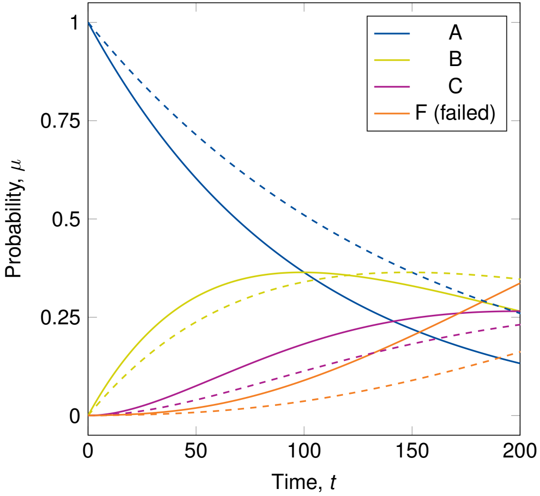

Figure: Solid line: $u = 1$.Dashed line: $u = 0.67$

Including maintenance interventions

How do we include maintenance interventions / inspections in this type of model?

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$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\Lambda}(u,t)\boldsymbol{\mu}(t) + \boldsymbol{R}\boldsymbol{r}(t) \tag{4}$$

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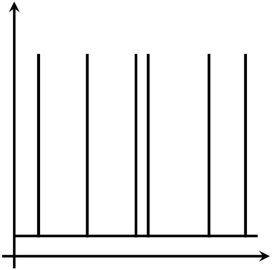


$\boldsymbol{R}\boldsymbol{r}(t)$ is a term that reset state to arbitrary condition

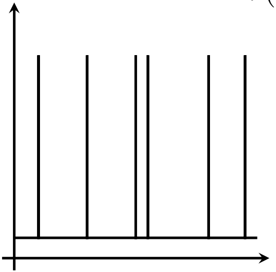
Including maintenance interventions

How do we include maintenance interventions / inspections in this type of model?

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\Lambda}(u,t)\boldsymbol{\mu}(t) + \boldsymbol{R}\boldsymbol{r}(t) \tag{4}$$

where

$$\boldsymbol{r}(t) = \boldsymbol{S}^{\mathsf{T}}\boldsymbol{\mu}(t) \cdot \left(\sum_{i=1}^{k} \delta(t - t_i)\right) \tag{5}$$



$$\boldsymbol{R} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tag{6}$$

$\boldsymbol{R}\boldsymbol{r}(t)$ is a term that reset state to arbitrary condition

As-good-as-new, corrective

$$\boldsymbol{R} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tag{7}$$

As-good-as-new, corrective

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As-good-as-new, preventive

$$\boldsymbol{R} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \tag{8}$$

As-good-as-new, corrective

$$\boldsymbol{R} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tag{7}$$

As-good-as-new, preventive

$$\boldsymbol{R} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \tag{8}$$

Imperfect, corrective

$$\boldsymbol{R} = \begin{bmatrix} 0.7 \\ 0.3 \\ 0 \\ -1 \end{bmatrix} \tag{9}$$

Including maintenance interventions

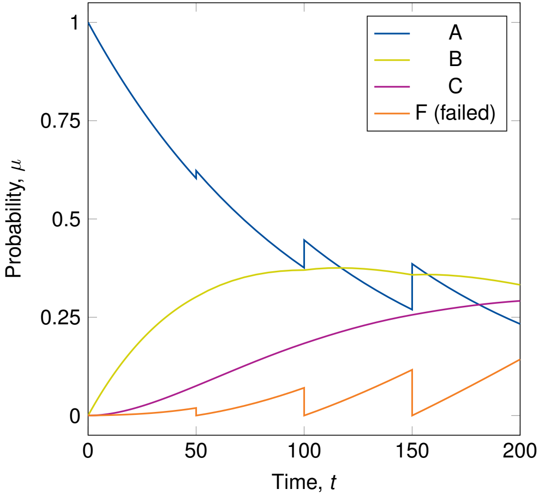

Figure: Evolution of the probabilities with inspections at 50, 100 and 150 weeks

Optimization problem

$$\min_{u(l),r(l)} \int_0^{t_f} \Big(-f + f_m + f_i \Big) dt \tag{10}$$

where

Optimization problem

$$\min_{u(l),r(l)} \quad \int_0^{t_f} \Big(- f + f_m + f_i \Big) dt \tag{10}$$

where

$$f(t) \quad = \quad \boldsymbol{c}(u,t)^{\mathsf{T}} \boldsymbol{\mu}(t) \qquad \text{Production}$$

Optimization problem

$$\min_{u(l),r(l)} \quad \int_0^{t_f} \Big(-f + f_m + f_i \Big) dt \qquad (10)$$

where

$$\begin{aligned} f(t) &= \boldsymbol{c}(u,t)^{\mathsf{T}} \boldsymbol{\mu}(t) && \text{Production} \\ f_m(t) &= \boldsymbol{c}_m^{\mathsf{T}} \boldsymbol{r}(t) && \text{Maintenance cost} \end{aligned}$$

Optimization problem

$$\min_{u(l),r(l)} \int_0^{t_f} \Big(-f + f_m + f_i \Big) dt \tag{10}$$

where

$$\begin{aligned} f(t) &= \boldsymbol{c}(u,t)^\intercal \boldsymbol{\mu}(t) && \text{Production} \\ f_m(t) &= \boldsymbol{c}_m^\intercal \boldsymbol{r}(t) && \text{Maintenance cost} \\ f_i(t) &= \boldsymbol{c}_i^\intercal \left(\sum_{i=1}^{k} \delta(t - t_i) \right) && \text{Inspection cost} \end{aligned}$$

Optimization problem

$$\min_{u(l),r(l)} \int_0^{t_f} \Big(-f + f_m + f_i \Big) dt \tag{10}$$

where

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Subject to

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\Lambda}(t,u)\boldsymbol{\mu}(t) + \boldsymbol{R}\boldsymbol{r}(t) \qquad \text{Model}$$

Solving the optimization problem

Our optimization problem is on the form:

$$\min_{u,r} \quad \int_0^{t_f} \phi(t, \mu, u, r) \tag{11}$$

$$\text{s.t.} \quad g(t, \mu, u, r) \leq 0 \tag{12}$$

Discretize problem, solve resulting nonlinear program (NLP) using off-the-shelf NLP solver

Optimal solution

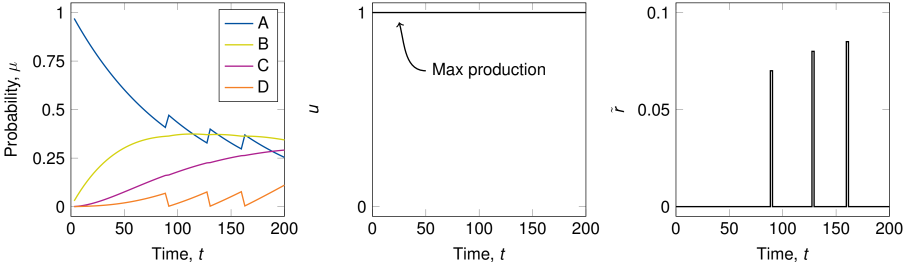
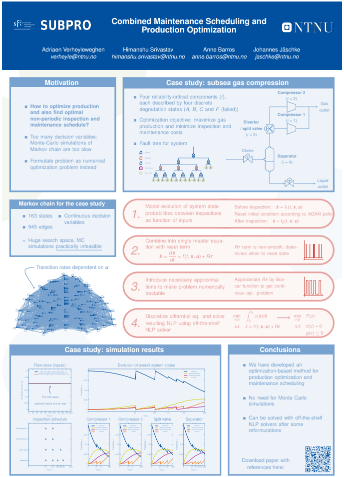


Figure: Optimal production strategy with maintenance after 92, 132 and 163 weeks.

Conclusion

- Can optimize maintenance and operations at the same time
- Would have been difficult with traditional Monte Carlo approach!



Future work:

- (more complex) non-linear model
- Distributionally robust
- Multi-stage maintenance planning

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Backup: Approximating r

Dirac functions are not good for optimization

$$\tilde{\boldsymbol{r}}(t) = \boldsymbol{S}^{\mathsf{T}}\boldsymbol{\mu}(t) \cdot \left(\sum_{i=1}^{k} \delta(t - t_i)\right) \tag{13}$$

$$\tilde{\boldsymbol{r}}(t) \approx \boldsymbol{r}(t) = \sum_{i=1}^{k} \mathsf{Boxcar}(t) = \sum_{i=1}^{k} h_i \Big(\mathsf{H}(t - t_i) - \mathsf{H}(t - t_i - \epsilon_i)\Big) \tag{14}$$

In practice, the Boxcar function is simply a piece-wise constant function

Backup: Approximating r

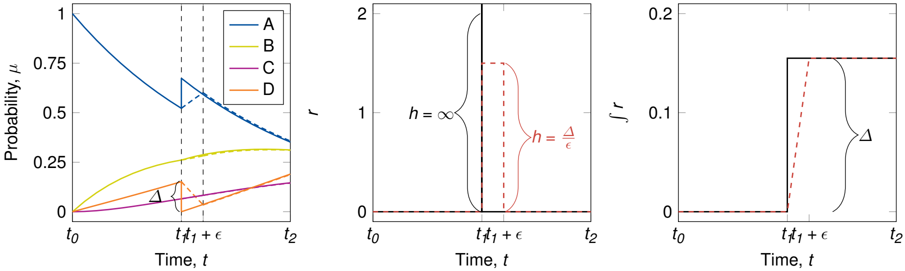

Figure: Illustration of how $\tilde{\boldsymbol{r}}(t)$ (dashed line, circles) can be approximated by $\boldsymbol{r}(t)$ (solid line) to obtain a continuous $\boldsymbol{\mu}$.

Backup: How to calculate f_i

Inspection cost is proportional to number of inspections. How to calculate?

Formulate with complementary

$$f_i = c_i \cdot y \tag{15}$$

$$0 \leq (1 - y) \perp r \geq 0 \tag{16}$$

Backup: How to calculate f_i

Can be solved in IPOPT with a hybrid relaxation / penalty method

$$\min_{\boldsymbol{\mu},\boldsymbol{r},\boldsymbol{u},y} \quad -f(\boldsymbol{\mu},\boldsymbol{r},\boldsymbol{u},t)+\boldsymbol{c}(l)\cdot \text{vec}\left((1-\boldsymbol{y}(t))\otimes \boldsymbol{u}(t)\right) \tag{17a}$$

$$\text{s.t.} \quad \frac{d\boldsymbol{\mu}}{dt}=\boldsymbol{\Lambda}\boldsymbol{\mu}+\boldsymbol{R}\boldsymbol{r} \tag{17b}$$

$$0\leq \boldsymbol{\mu}(t)\leq 1 \tag{17c}$$

$$0\leq \boldsymbol{r}(t)\leq \bar{\boldsymbol{r}} \tag{17d}$$

$$0\leq \boldsymbol{u}(t)\leq \bar{\boldsymbol{u}} \tag{17e}$$

$$\text{vec}\left(\boldsymbol{r}(t)\otimes(1-\boldsymbol{y}(t))\right)\geq \boldsymbol{\epsilon}(l). \tag{17f}$$

Here, $(\cdot)$ denotes the inner product and $(\otimes)$ denotes the outer product. This problem is solved sequentially $\{l\}$ times such that

$$\lim_{l\to\infty}, \boldsymbol{\epsilon}\to 0, \boldsymbol{c}\to\infty$$