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A Multistage Economic Model Predictive Control for Time-Varying Parameter Value

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Objective

In this work we propose:

- one-layer economic model predictive control (EMPC), eliminating the need of real-time optimization (RTO),
- fast update scheme to multistage nonlinear EMPC for dealing with parametric uncertainties, based on nonlinear programming (NLP) sensitivity concept.

We design an EMPC controller for controlling a system that evolves according to the following equation

$$\mathbf{x}_{k+1} = f\left(k, \mathbf{x}_k, \mathbf{u}_k, \boldsymbol{\theta}_k\right), \qquad (1)$$

where $k \in \mathbb{N}_0$ represents a time instant, $\mathbf{x} \in \mathbb{R}^{n_x}$ denotes the state, $\mathbf{u} \in \mathbb{R}^{n_u}$, and where $\boldsymbol{\theta} \in \mathbb{R}^{n_{\boldsymbol{\theta}}}$ denotes a time-varying parameter vector. Predictions of the parameter $\boldsymbol{\theta}$ are known.

Multistage EMPC formulation

We consider a dynamic optimization problem

$$\begin{aligned} \mathcal{P}_N\left(\mathbf{x}_k\right): \min_{\mathbf{x}_{s,j},\mathbf{u}_{s,j},\mathbf{z}_{l,j},\mathbf{v}_{l,j}} & \sum_{j=1}^{n_S} \omega_j \left(\sum_{l=0}^{N-1} \psi\left(\mathbf{z}_l, \mathbf{v}_l, \boldsymbol{\theta}_l\right)\right) \\ s.t. \quad & f\left(N, \mathbf{x}_{s,j}, \mathbf{u}_{s,j}, \boldsymbol{\theta}_{N,j}\right) = 0, \\ & \mathbf{z}_{l+1,j} = f\left(l, \mathbf{z}_{l,j}, \mathbf{v}_{l,j}, \boldsymbol{\theta}_{l,j}\right), \\ & \mathbf{z}_{0,j} = \mathbf{x}_k, \\ & \textstyle\sum_{j=1}^{n_S} \mathbf{E}_j \mathbf{v}_j = 0, \\ & \left(\mathbf{z}_{l,j}, \mathbf{v}_{l,j}\right) \in \mathcal{Z}, \\ & l = 0, \ldots, N-1, \\ & j = 1, \ldots, n_S. \end{aligned} \qquad (2)$$

Variable	Description
$\mathbf{x}_{s,j}$	artificial steady-state state for scenario j
$\mathbf{u}_{s,j}$	artificial control for scenario j
l	time instance
$\mathbf{z}_{l,j}$	predicted state for scenario j
$\mathbf{v}_{l,j}$	predicted control for scenario j
$\boldsymbol{\theta}_{l,j}$	time-varying parameter for scenario j
ω_j	probability for each scenario
n_S	number of scenario and prediction horizon
N	prediction horizon
$f\left(l, \mathbf{z}_{l,j}, \mathbf{v}_{l,j}, \boldsymbol{\theta}_{l,j}\right)$	prediction model
$\mathbf{z}_{0,j}$	initial condition for predicted state
$\mathbf{x}_k$	measurement at time k
$\mathbf{E}$	matrix of non-anticipativity contraint
$\mathcal{Z}$	path constraints for state and control

Path-following Multistage NMPC (pf-msNMPC)

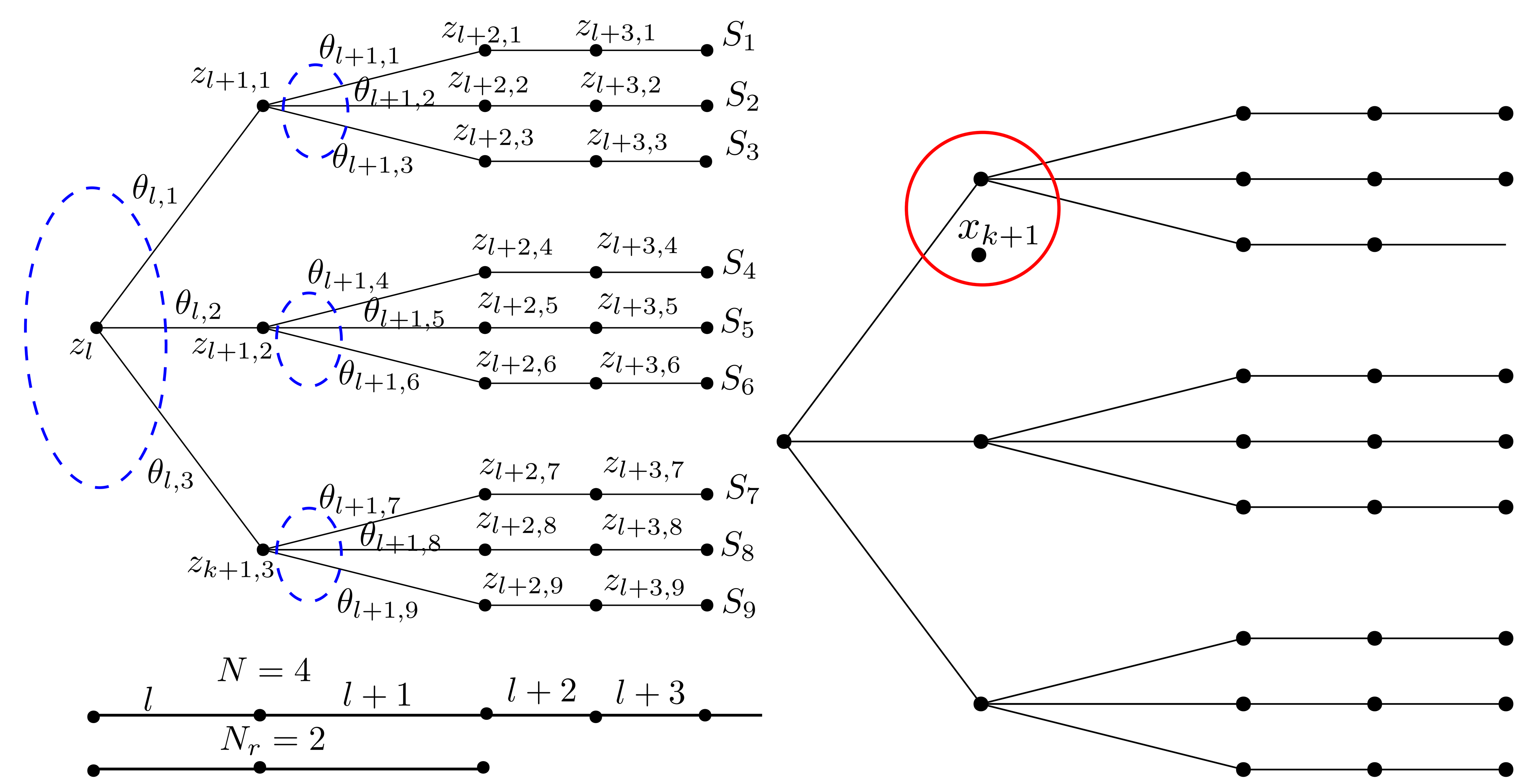

Fig. 1: Left: the parametric uncertainties are realized as a scenario tree, where the blue colors represent the NACs. Right: when a measurement available at $k+1$. The pf-msNMPC controller chooses a scenario that is nearest to the measurement, and update the scenario by solving a sequence of predictor-corrector quadratic programs.

A Case Example

We implement the EMPC controller for a CSTR with first order reaction $A \rightarrow B$

$$\begin{aligned} \frac{dc_A}{dt} &= \frac{Q}{V}\left(c_{Af} - c_A\right) - kc_A \\ \frac{dc_B}{dt} &= \frac{Q}{V}\left(-c_B\right) - kc_A, \end{aligned} \qquad (3)$$

where c_A and c_B are the concentration of components A and B, respectively.

- The rate constant is $k = 1.2 \frac{L}{mol\, minute}$, the reactor volume is $V = 10\, L$, and the feed concentration is $c_{Af} = 1 \frac{mol}{L}$.
- The control input is denoted by Q with unit $\frac{L}{minute}$ and the state variables are the concentration c_A and c_B.
- The economic objective function is $L\left(c_A, c_B, Q\right) = -Q\left(p_C\, c_B - p_Q\right)$, where p_C and p_Q are product price and material cost
- Bound constraints on the control input $10 \leq Q \leq 20$
- Modified stage cost $\psi_m := -Q\left(p_C\, c_B - p_Q\right) + 2\left(Q - u_s\right)^2$, where u_s is the artificial control input
- The varying parameters here are the product price (p_C) and material cost (p_Q)
- The parameters change their values at first, 25th, and 80th EMPC iterations
- Prediction horizon for EMPC is $N = 50$
- Multistage EMPC with robust horizon, $N_r = 1$, and 4 different scenarios that allow $\pm 25\%$, $\pm 50\%$, $\pm 100\%$, and $\pm 125\%$ variations in the prices.

Simulation Results

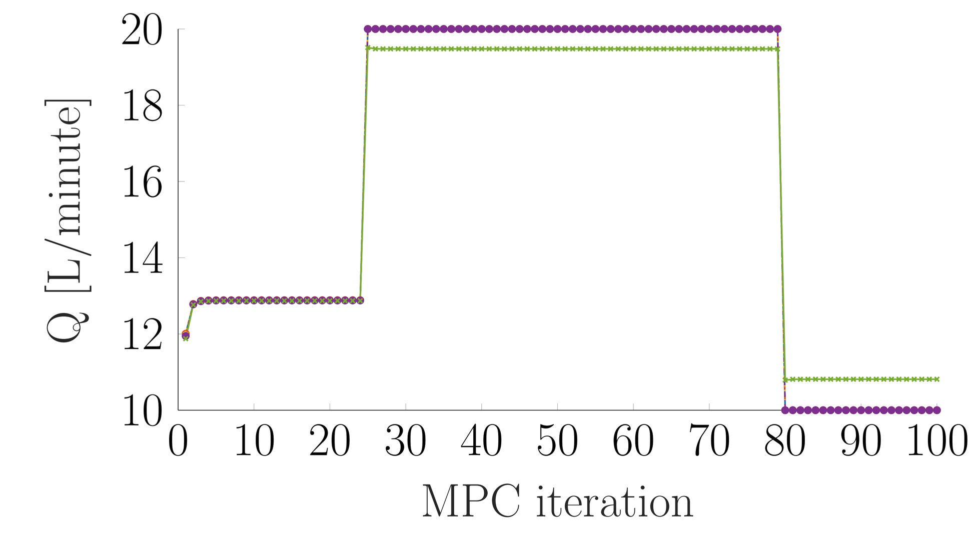

Fig. 2: Optimized control input with regularization term with the colors represent: nominal MPC (-), multistage MPC ±25% (-), multistage MPC ±50% (-), multistage MPC ±100% (-), multistage MPC ±125% (-)

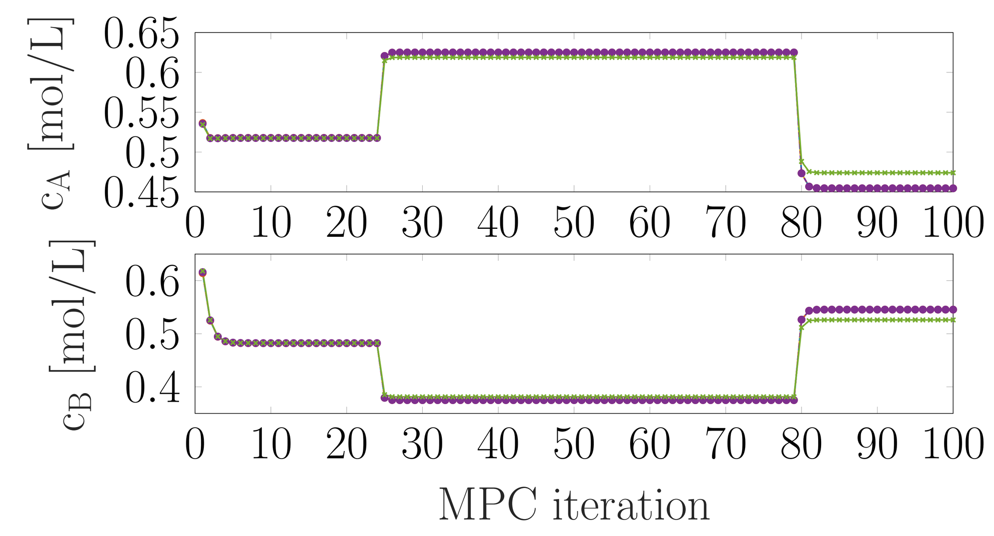

Fig. 3: Optimized state variables with regularization term.

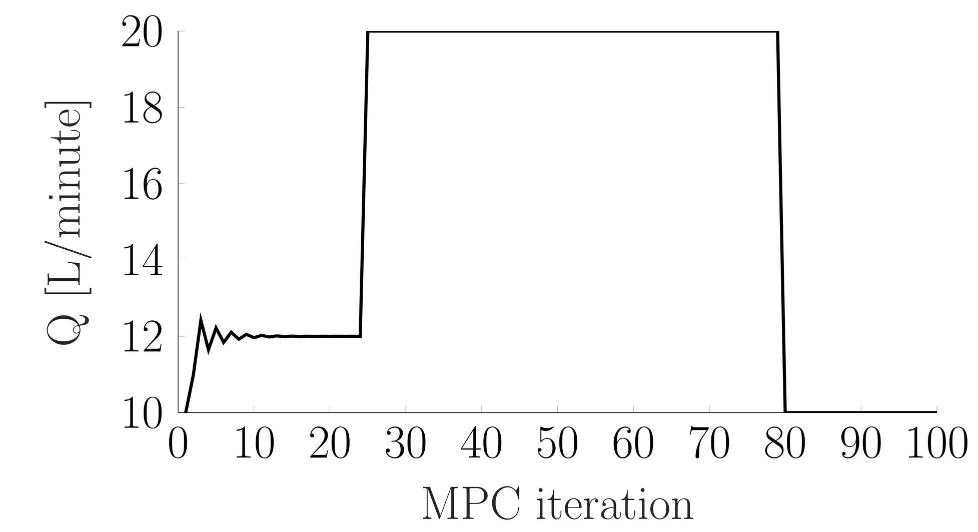

Fig. 4: Optimized control input without regularization term.

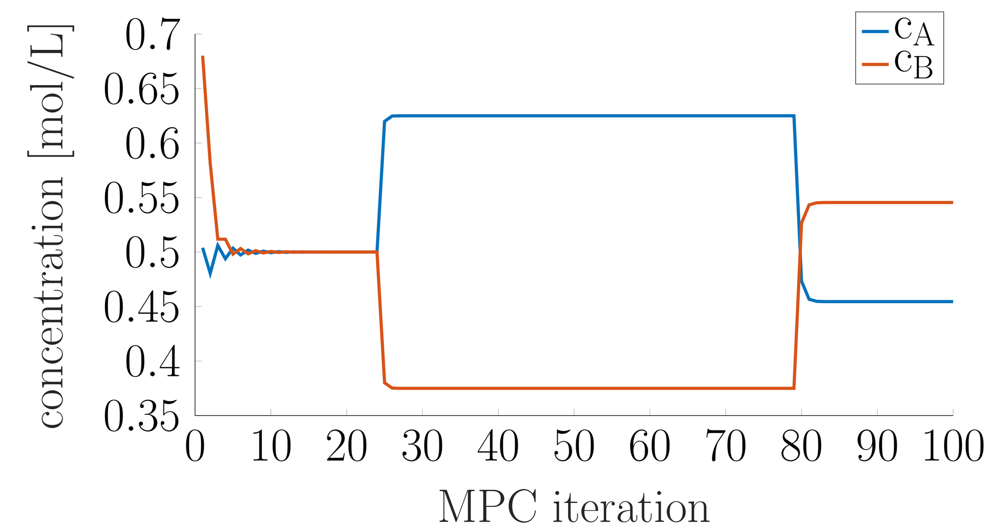

Fig. 5: Optimized state variables without regularization term.

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