Fast Economic Model Predictive Control for a Gas Lifted Well Network ⋆

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Abstract: This paper considers the optimal operation of oil and gas production networks by formulating it as an economic nonlinear model predictive control (ENMPC) problem. Solving nonlinear dynamic optimization for large-scale systems is often computationally expensive, which is a prohibitive factor in many applications. In this paper, we explicitly address the computational issues associated with economic NMPC problems for large-scale systems. We apply an NLP sensitivity update based on a path-following predictor-corrector algorithm for fast implementation of economic NMPC, which approximates the NLP solution using a sequence of quadratic programs. We demonstrate the proposed method and compare the performance of the path-following economic NMPC with the standard economic NMPC problem using gas-lift optimization problem as our case study.

Keywords: sensitivity-based NMPC, path-following algorithm, dynamic optimization, production optimization, gas-lift optimization

1. INTRODUCTION

The optimal operation of an oil and gas production network involves taking various decisions in order to maximize the daily operating revenue while satisfying process and operating constraints. This is known as short-term production optimization or daily production optimization. Mathematical tools are often used in production optimization to compute the optimal decision variables based on a mathematical representation of the system. The use of mathematical optimization for daily production optimization has reported increases in the range of 1-4% (Stenhouse et al., 2010). A comprehensive survey of optimization tools used for production optimization can be found in Bieker et al. (2007). DPO problems may be formulated as either static optimization problems or dynamic optimization problems. Useful discussions on the static and dynamic formulations for DPO are provided in Foss et al. (2017), where the authors note that DPO applications may benefit from dynamic formulations in some cases. Particularly, where the link to lower level control is important.

In this paper, we consider such a dynamic production optimization problem, where the control and optimization layers are tightly integrated. In other words, the production optimization problem is formulated as an economic nonlinear model predictive control (ENMPC) problem, which has been receiving tremendous attention recently. The key idea in economic model predictive control is to use a single dynamic optimization problem to control and to optimize the economic performance. By doing so, the economic cost is optimized during transient operation of the system. In the face of volatile oil prices and competitive market, optimizing during the transients to maximize profits has become more and more desirable.

For large scale production networks, the economic NMPC problem formulation may be rather large with several hundred decision variables. For example, the production optimization of the Troll oil field in the Norwegian continental shelf includes more than one hundred subsea wells (Hauge et al., 2005). This leads to the optimization problem being very computationally intensive. Additionally, nonlinear models are typically used for economic optimization, which further adds on to the computational complexity. Computational cost has been a prohibitive factor in the widespread implementation of dynamic optimization in the oil and gas industry despite being a promising approach as noted in Forbes et al. (2015). Solving the large scale NLP may take significant amount of time. This computational delay can potentially lead to performance degradation or even to closed-loop instabilities (Findeisen and Allgöwer, 2004). Hence there is a clear need to develop tools to enable real-time implementation of such dynamic optimization methods.

Several sensitivity-based approaches have been proposed to address this issue, see e.g. Diehl et al. (2005), Zavala and Biegler (2009), Jäschke et al. (2014) and a review article on fast NMPC schemes in Wolf and Marquardt (2016). At each sample time, the NMPC optimization problems are identical, except for one time varying parameter, namely, the initial state. All the fast sensitivity approaches capitalize on this property. When new measurements of the states become available, these approaches use the sensitivity of the NLP solution that was computed at the previous time step to obtain fast approximate solutions to the nonlinear optimization problem. Such approximations

⋆ The authors gratefully acknowledge the financial support from Research Council of Norway through IKTPLUSS Young Researcher Grant and SFI SUBPRO programs.

enable fast computation and implementation in the plant with minimal delay. In this paper, we apply the fast sensitivity-based economic NMPC as described by Jäschke et al. (2014) and Suwartadi et al. (2017), named *path-following method*, to a gas-lift optimization problem, which allows large changes in parameter values.

The reminder of the paper is organized as follows. The daily production optimization problem for a gas lifted well network is formalized in section 2. The sensitivity-based economic NMPC with the path-following approach is presented in section 3. The simulation results from a gas lift optimization example is presented in section 4 before concluding the paper in section 5.

2. PROBLEM FORMULATION

An offshore oil and gas production network typically consists of several wells that produce to a common processing facility. The reservoir fluid enters through the well bore of each well and produce to a topside processing facility via a common production manifold as shown in Fig.1. In some wells, the reservoir pressure may not be sufficient to lift the fluids to the surface economically. In such cases, artificial lift methods are employed to boost the production from the wells. In this paper, we consider gas lift method which is a commonly used artificial left technology. In gas lifted wells, compressed gases are injected in the well tubing via the annulus to reduce the mixture density. This results in reduced hydrostatic pressure losses hence boosting production. However, increased gas injection also increases frictional pressure drop. The oil production starts to decline if the effect of the frictional pressure drop is dominant over the effect of the hydrostatic pressure drop.

Production from a cluster of $\mathcal{N} = \{1, \ldots, n_w\}$ gas lifted well can be modelled as a semi-explicit index-1 DAE system of the form,

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{z}, \mathbf{u}), \tag{1a}$$

$$0 = g(\mathbf{x}, \mathbf{z}, \mathbf{u}), \tag{1b}$$

where $f(\mathbf{x}, \mathbf{z}, \mathbf{u})$ is the set of differential equations and $g(\mathbf{x}, \mathbf{z}, \mathbf{u})$ is the set of algebraic equations. The dynamics arise in the model due to the mass balances for oil and gas phases in each well and the riser. Algebraic equations are used to describe the densities, pressures and flow rates for each well and the riser as described in detail in Krishnamoorthy et al. (2016).

The inlet separator in the topside processing facility sets the downstream boundary conditions which are typically kept at a constant pressure by tight regulatory control. The upstream boundary conditions are set by the reservoir inflow conditions. The DPO problem is concerned with the production network exposed to these upstream and downstream conditions and hence the reservoir model and topside processing facilities are not included in the production optimization problem.

Gas lifted wells are often controlled by adjusting the gas lift injection rate of each well. The production network is also generally subject to process and operating constraints. For example, the total gas processing capacity in the topside facilities may be constrained, or the total compressed gas for gas lift may be limited. The optimization problem then involves computing the optimal gas lift injection rates

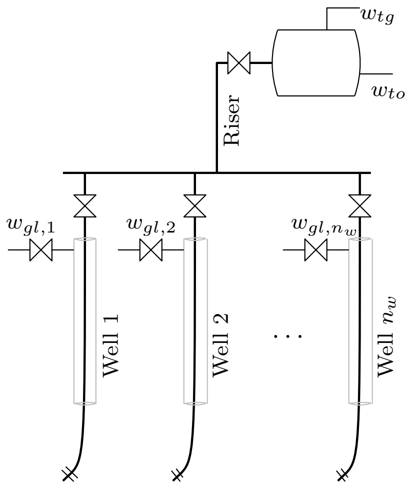

Fig. 1. Schematic representation of a gas lifted production network with n_w wells producing to a common topside processing facility.

for each well such the operating profits are maximized subject to the network processing and operating constraints.

Before this can be formalized as an economic NMPC problem, the infinite dimensional problem is discretized into finite horizon optimal control problem (FHOCP) using direct collocation method for the system (1a). By using direct collocation method, we avoid numerical convergence issue during the integration of dynamical system. The discretized system dynamics at any time instant l can be expressed as,

$$\mathbf{F}\left(\boldsymbol{\chi}_{l+1}, \boldsymbol{\chi}_l, \boldsymbol{\zeta}_l, \boldsymbol{\nu}_l\right) = 0 \tag{2}$$

A detailed explanation on how the system is discretized into a nonlinear programming problem using direct collocation can be found in Krishnamoorthy et al. (2016).

The economic NMPC problem can then be formalized,

$$\begin{aligned}
\mathcal{P}_N\left(\mathbf{x}_k\right) : \min_{\boldsymbol{\chi}_l, \boldsymbol{\nu}_l, \boldsymbol{\zeta}_l} & \Psi\left(\boldsymbol{\chi}_{k+N}, \boldsymbol{\nu}_{k+N}\right) + \sum_{l=k}^{k+N-1} \psi\left(\boldsymbol{\chi}_l, \boldsymbol{\nu}_l, \boldsymbol{\zeta}_l\right) \\
\text{s.t. } & \mathbf{F}\left(\boldsymbol{\chi}_{l+1}, \boldsymbol{\chi}_l, \boldsymbol{\zeta}_l, \boldsymbol{\nu}_l\right) = 0, \quad \forall l \in \mathcal{N} \\
& \mathbf{G}\left(\boldsymbol{\chi}_l, \boldsymbol{\zeta}_l, \boldsymbol{\nu}_l\right) \leq 0, \quad \forall l \in \mathcal{N} \\
& \left(\boldsymbol{\chi}_l, \boldsymbol{\nu}_l, \boldsymbol{\zeta}_l\right) \in \mathcal{Z}, \quad \forall l \in \mathcal{N} \\
& \left(\boldsymbol{\chi}_{k+N}, \boldsymbol{\nu}_{k+N}\right) \in \mathcal{X}_f \\
& \boldsymbol{\chi}_k = \mathbf{x}_k,
\end{aligned} \tag{3}$$

Here $\boldsymbol{\chi}_l \in \mathbb{R}^{n_\chi}$, $\boldsymbol{\nu}_l \in \mathbb{R}^{n_\nu}$, and $\boldsymbol{\zeta}_l \in \mathbb{R}^{n_\zeta}$ represent the predicted state, algebraic, and control input at time instance l, respectively for all l belonging to $\mathcal{N} = \{k, \ldots, k+N\}$. The constraints include the system dynamics posed as equality constraint, nonlinear inequality constraints $\mathbf{G}$, the equality constraint of the initial predicted state $\boldsymbol{\chi}_0$ equal to the actual state $\mathbf{x}_k \in \mathbb{R}^{n_\chi}$ obtained from measurement data, the path constraint confines the predicted state, algebraic, and control inside the set $\mathcal{Z}$, and the final state and algebraic variables $\left(\boldsymbol{\chi}_{k+N}, \boldsymbol{\nu}_{k+N}\right)$ lie in the set $\mathcal{X}_f$. The objective function comprises the final cost $\Psi\left(\boldsymbol{\chi}_{k+N}, \boldsymbol{\nu}_{k+N}\right) \in \mathcal{C}^2 : \mathbb{R}^{n_\chi} \times \mathbb{R}^{n_\nu} \to \mathbb{R}$ and the stage cost $\psi\left(\boldsymbol{\chi}_l, \boldsymbol{\nu}_l, \boldsymbol{\zeta}_l\right) \in \mathcal{C}^2 : \mathbb{R}^{n_\chi} \times \mathbb{R}^{n_\nu} \times \mathbb{R}^{n_\zeta} \to \mathbb{R}$.

In the gas lifted well problem, the stage cost is defined as

$$\psi\left(\boldsymbol{\chi}_l, \boldsymbol{\nu}_l, \boldsymbol{\zeta}_l\right) := \sum_{i=1}^{n_w} \left(-r_o w_{po,i} + r_{gl} w_{gl,i}\right) \tag{4}$$

where r_o is the oil price, r_{gl} is the cost for gas lift injection, and n_w denotes the number of production well. The nonlinear inequality constraints enforce the total gas capacity constraints,

$$\mathbf{G}\left(\boldsymbol{\chi}_l, \boldsymbol{\zeta}_l, \boldsymbol{\nu}_l\right) := \sum_{i=1}^{n_w} \left(w_{pg,i}\right) - w_g^{max} \tag{5}$$

The path constraints are in the form of bound constraints for the state, algebraic variables as well as the control inputs

$$\begin{pmatrix} \underline{\boldsymbol{\chi}} \\ \underline{\boldsymbol{\nu}} \\ \underline{\boldsymbol{\zeta}} \end{pmatrix} \leq \begin{pmatrix} \boldsymbol{\chi}_l \\ \boldsymbol{\nu}_l \\ \boldsymbol{\zeta}_l \end{pmatrix} \leq \begin{pmatrix} \overline{\boldsymbol{\chi}} \\ \overline{\boldsymbol{\nu}} \\ \overline{\boldsymbol{\zeta}} \end{pmatrix} \tag{6}$$

The notations $\bar{\cdot}$ and $\underline{\cdot}$ represent the upper and lower bound for the corresponding variable. It is possible to limit the variables with the bound constraints since the system dynamics is transcribed using direct collocation method in which the state, algebraic, and control are considered as optimization variables.

Once the economic NMPC problem is formalized, it can be solved in a receding horizon fashion. At each sample time k, the state measurement or estimate $\mathbf{x}_k$ is assigned as the initial state for the optimization problem (3). The optimization problem is solved to compute the optimal input trajectory $\boldsymbol{\zeta}^*_{[l,l+N]}$ over the prediction horizon. The first step of the optimal control sequence is implemented on the plant, i.e. $\mathbf{u}_k^* = \boldsymbol{\zeta}_1^*$. At the next time step $k+1$, new measurements of the state $\mathbf{x}_{k+1}$ are obtained and the optimization procedure is repeated, hence enabling closed-loop implementation.

We consider a full-state feedback control structure. The measurements from the plant are used for estimating the states. The estimated states are then used for full state feedback for the economic NMPC, which computes the optimal inputs for the plant. The state estimation is performed online with the help of an extended Kalman filter (EKF). The EKF is implemented in discrete time as described in Krishnamoorthy et al. (2017). Commonly available measurements such as annulus pressure, wellhead pressure and down hole pressure for each well along with the riser head pressure, manifold pressure and total oil and gas production rates were used as available measurements for the EKF.

In an ideal case, the optimization problem (3) is solved instantly and the optimal input is implemented on the plant without time delay. In practice, however, there is always some delay between the state measurement/estimation and the implementation of the optimal control input. This is mainly because of the computational time required to solve the optimization problem (3). For many linear MPC applications, the computational delay is rather small and can be neglected. However, for large-scale nonlinear systems, as the number of optimization variable increases, solving the optimization problem requires more time, and the computational delay may no longer be neglected.

3. FAST ECONOMIC NMPC

To address the issue of computational delay, fast sensitivity-based NMPC approaches have been developed. One such approached is the advanced-step NMPC (asNMPC) as introduced by Zavala and Biegler (2009), where at time k, the NMPC problem is solved with the predicted state value of time $k+1$ instead of using the measured/estimated state $\mathbf{x}_k$. An approximation of the NLP problem is then computed using a single sensitivity step (Zavala and Biegler, 2009), or a path-following approach (Jäschke et al., 2014; Suwartadi et al., 2017). when the measurements $\mathbf{x}_{k+1}$ is available at time $k+1$. The proposed predictor-corrector path-following algorithm is based on Suwartadi et al. (2017) and is described below.

Since the optimization problem (3) differs only in the initial state variable $\boldsymbol{\chi}_0$ from one NMPC iteration to another, the problem can be cast as the following parametric NLP problem

$$\begin{aligned} \min_{\boldsymbol{X}} \quad & \mathcal{J}\left(\boldsymbol{X}, \boldsymbol{p}\right) \\ \text{s.t. } & c_i\left(\boldsymbol{X}, \boldsymbol{p}\right) = 0, i \in \mathcal{E}, \\ & c_i\left(\boldsymbol{X}, \boldsymbol{p}\right) \leq 0, i \in \mathcal{I}, \end{aligned} \tag{7}$$

where $\boldsymbol{X} \in \mathbb{R}^{n_{\boldsymbol{X}}}$ is the primal variable, $\boldsymbol{p} \in \mathbb{R}^{n_{\boldsymbol{p}}}$ is the parameter, and the objective function $\mathcal{J} : \mathbb{R}^{n_{\boldsymbol{X}}} \times \mathbb{R}^{n_{\boldsymbol{p}}} \longrightarrow \mathbb{R}$. The equality and inequality constraints $c : \mathbb{R}^{n_{\boldsymbol{X}}} \times \mathbb{R}^{n_{\boldsymbol{p}}} \longrightarrow \mathbb{R}^{n_c}$ are represented by the sets $\mathcal{E} = \{1, \ldots, m\}$ and $\mathcal{I} = \{m+1, \ldots, n\}$, respectively.

We define Lagrangian of the problem (7) as

$$\mathcal{L}\left(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{p}\right) := \mathcal{J}\left(\boldsymbol{X}, \boldsymbol{p}\right) + \boldsymbol{\lambda}^T c\left(\boldsymbol{X}, \boldsymbol{p}\right), \tag{8}$$

where $\boldsymbol{\lambda}$ is the dual variable or Lagrange multiplier. Moreover, the first-order optimality (*Karush-Kuhn-Tucker (KKT)*) conditions are

$$\begin{aligned} \nabla_{\boldsymbol{X}} \mathcal{L}(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{p}) &= 0, \\ c_i\left(\boldsymbol{X}, \boldsymbol{p}\right) &= 0,\ i \in \mathcal{E}, \\ c_i\left(\boldsymbol{X}, \boldsymbol{p}\right) &\leq 0,\ i \in \mathcal{I}, \\ \boldsymbol{\lambda}_i^T c_i\left(\boldsymbol{X}, \boldsymbol{p}\right) &= 0,\ i \in \mathcal{I}, \\ \boldsymbol{\lambda}_i &\geq 0,\ i \in \mathcal{I}. \end{aligned} \tag{9}$$

Active inequality constraints are denoted by the set $\mathcal{A}\left(\boldsymbol{X}, \mathbf{p}\right) = \{c_i\left(\boldsymbol{X}, \mathbf{p}\right) = 0, i \in \mathcal{I}\}$. For a given multiplier $\boldsymbol{\lambda}$ and $\boldsymbol{X}^*$ that satisfies the KKT condition (9), the active inequality set $\mathcal{A}\left(\boldsymbol{X}, \mathbf{p}\right)$ has two subsets, which are a weakly active set $\mathcal{A}_0\left(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{p}\right) = \{i \in \mathcal{A}\left(\boldsymbol{X}, \boldsymbol{p}\right) \mid \boldsymbol{\lambda}_i = 0\}$ and a strong active set $\mathcal{A}_+\left(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{p}\right) = \{i \in \mathcal{A}\left(\boldsymbol{X}, \boldsymbol{p}\right) \mid \boldsymbol{\lambda}_i > 0\}$.

Furthermore, we define the residual optimality as

$$\eta\left(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{p}\right) = \left\| \begin{pmatrix} \nabla_{\boldsymbol{X}} \mathcal{J}\left(\boldsymbol{X}, \boldsymbol{p}\right) + \nabla_{\boldsymbol{X}} c\left(\boldsymbol{X}, \boldsymbol{p}\right) \boldsymbol{\lambda} \\ c\left(\boldsymbol{X}, \boldsymbol{p}\right)_{\mathcal{E}} \\ \left[\min\left(c\left(\boldsymbol{X}, \boldsymbol{p}\right), \boldsymbol{\lambda}\right)\right]_{\mathcal{I}} \end{pmatrix} \right\|_{\infty}. \tag{10}$$

Here, we assume satisfaction of the linear independent constraint qualification (LICQ), which is defined as follows.

Definition 1. (LICQ). Given a vector $\boldsymbol{p}$ and a point $\boldsymbol{X}$, the *linear independence constraint qualification* (LICQ) holds at $\boldsymbol{X}$ if the set of vectors $\{\nabla_{\boldsymbol{X}} c_i\left(\boldsymbol{X}, \boldsymbol{p}\right)\}_{i \in \mathcal{E} \cup \mathcal{A}(\boldsymbol{X}, \boldsymbol{p})}$ are linearly independent.

We also assume that the strong second order sufficient condition is also satisfied.

Definition 2. (SSOSC). The *strong second order sufficient condition* (SSOSC) holds at $\boldsymbol{X}$ and multipliers $\boldsymbol{\lambda}$, which are satisfying the KKT conditions, if $\boldsymbol{d}^T \nabla^2_{\boldsymbol{X}\boldsymbol{X}} \mathcal{L}(\boldsymbol{X}, \boldsymbol{p}, \boldsymbol{\lambda}) \boldsymbol{d} > 0$ for all $\boldsymbol{d} \neq 0$ such that $\nabla_{\boldsymbol{X}} c_i (\boldsymbol{X}, \boldsymbol{p})^T \boldsymbol{d} = 0$ for $i \in \mathcal{E} \cup \mathcal{A}_+ (\boldsymbol{X}, \boldsymbol{p}, \boldsymbol{\lambda})$.

We are now ready to state the result for sensitivity of the NLP, where $\boldsymbol{X}^* (\boldsymbol{p})$ and $\boldsymbol{\lambda}^* (\boldsymbol{p})$ are the primal and dual solutions of (7), respectively.

Theorem 3. Let $\mathcal{J}$, c be twice continuously differentiable in $\boldsymbol{p}$ and $\boldsymbol{X}$ near a solution of (7) $(\boldsymbol{X}^*, \boldsymbol{p}_0)$, and let LICQ and SSOSC hold at $(\boldsymbol{X}^*, \boldsymbol{p}_0)$. Then the solution $(\boldsymbol{X}^* (\boldsymbol{p}), \boldsymbol{\lambda}^* (\boldsymbol{p}))$ is Lipschitz continuous in a neighborhood of $(\boldsymbol{X}^*, \boldsymbol{\lambda}^*, \boldsymbol{p}_0)$, and the solution function $(\boldsymbol{X}^* (\boldsymbol{p}), \boldsymbol{\lambda}^* (\boldsymbol{p}))$ is directionally differentiable. Moreover, the directional derivative uniquely solves the following quadratic problem:

$$\begin{aligned}
\min_{\Delta \boldsymbol{X}} \quad & \frac{1}{2} \Delta \boldsymbol{X}^T \nabla^2_{\boldsymbol{X}\boldsymbol{X}} \mathcal{L}(\boldsymbol{X}^*, \boldsymbol{p_0}, \boldsymbol{\lambda}^*) \Delta \boldsymbol{X} \\
& + \Delta \boldsymbol{X}^T \nabla_{\boldsymbol{p}\boldsymbol{X}} \mathcal{L}(\boldsymbol{X}^*, \boldsymbol{p_0}, \boldsymbol{\lambda}^*) \Delta \boldsymbol{p} \qquad (11)\\
\text{s.t.} \quad & \\
& c_i (\boldsymbol{X}^*, \boldsymbol{p_0}) + \nabla_{\boldsymbol{X}} c_i (\boldsymbol{X}^*, \boldsymbol{p_0})^T \Delta \boldsymbol{X} \\
& + \nabla_{\boldsymbol{p}} c_i (\boldsymbol{X}^*, \boldsymbol{p_0})^T \Delta \boldsymbol{p} = 0 \qquad i \in \mathcal{A}_+ \cup \mathcal{E}, \\
& c_j (\boldsymbol{X}^*, \boldsymbol{p_0}) + \nabla_{\boldsymbol{X}} c_j (\boldsymbol{X}^*, \boldsymbol{p_0})^T \Delta \boldsymbol{X} \\
& + \nabla_{\boldsymbol{p}} c_j (\boldsymbol{X}^*, \boldsymbol{p_0})^T \Delta \boldsymbol{p} \leq 0 \qquad j \in \mathcal{A}_0
\end{aligned}$$

Proof. See Robinson (1980) and (Bonnans and Shapiro, 1998, Section 5.2). □

The theorem above implies that, instead of solving a full NLP problem, one can solve a quadratic programming (QP) problem (11) to compute the solution of the optimization problem (3) in the vicinity of perturbation $\boldsymbol{p}_0$. We refer the QP (11) to as *pure-predictor QP*. Note that there is no requirement of strict complimentary in the theorem allowing an active-set change occurs.

In order to improve the approximation solution, we introduce a corrector term in the objective function and by take into account that the parameter $\boldsymbol{p}$ enter linearly to the problem so that the QP formulation becomes (Suwartadi et al., 2017; Kungurtsev and Diehl, 2014)

$$\begin{aligned}
\min_{\Delta \boldsymbol{X}} \quad & \frac{1}{2} \Delta \boldsymbol{X}^T \nabla^2_{\boldsymbol{X}\boldsymbol{X}} \mathcal{L}(\boldsymbol{X}^*, \boldsymbol{p}_0 + \Delta \boldsymbol{p}, \boldsymbol{\lambda}^*) \Delta \boldsymbol{X} \\
& + \nabla_{\boldsymbol{X}} \mathcal{J}^T \Delta \boldsymbol{X} \qquad (12)\\
\text{s.t.} \quad & \\
& c_i (\boldsymbol{X}^*, \boldsymbol{p}_0 + \Delta \boldsymbol{p}) + \nabla_{\boldsymbol{p}} c_i (\boldsymbol{X}^*, \boldsymbol{p}_0 + \Delta \boldsymbol{p})^T \Delta \boldsymbol{p} + \\
& \nabla_{\boldsymbol{X}} c_i (\boldsymbol{X}^*, \boldsymbol{p}_0 + \Delta \boldsymbol{p})^T \Delta \boldsymbol{X} = 0 \ , i \in \mathcal{A}_+ \cup \mathcal{E}, \\
& c_j (\boldsymbol{X}^*, \boldsymbol{p}_0 + \Delta \boldsymbol{p}) + \nabla_{\boldsymbol{p}} c_j (\boldsymbol{X}^*, \boldsymbol{p}_0 + \Delta \boldsymbol{p})^T \Delta \boldsymbol{p} \\
& + \nabla_{\boldsymbol{X}} c_j (\boldsymbol{X}^*, \boldsymbol{p}_0 + \Delta \boldsymbol{p})^T \Delta \boldsymbol{X} \leq 0 \ , j \in \mathcal{A}_0.
\end{aligned}$$

We denote the QP formulation above as *predictor-corrector QP*. This QP formulation provides a reasonably good approximation of an NLP solution in the small neighborhood of $\boldsymbol{p}_0$. To allow large perturbation (large $\Delta \boldsymbol{p}$), we employ path-following approach, that is, to solve a series of QP problems. This is analogous to Euler integration scheme for ordinary differential equations. The parameter $\boldsymbol{p}$ is updated according to $\boldsymbol{p}(t_k) = (1 - t_k) \boldsymbol{p}_0 + t_k \boldsymbol{p}_f$, where $t_0 = 0$ until it reaches $t_k = 1$, that is $t_0 = 0 < t_1 < t_2 \ldots < t_k = 1$. The parameter $\boldsymbol{p}_f$ corresponds to new measurement data. During the course of path-following iteration, the solutions include primal variable $\Delta \boldsymbol{X}$ and dual variable $\Delta \boldsymbol{\lambda}$. The residual optimality condition $\eta_{k+\Delta}$ is computed and compared against its maximum tolerance η_{max}. This gives an update to the step size Δt. The method is implemented as a subroutine in Algorithm 1, denoted as `QP_PC_PF`.

As described in Suwartadi et al. (2017) and Jäschke et al. (2014), we apply the path-following QP within the advanced-step NMPC (*asNMPC*) framework (see Zavala and Biegler (2009)) and refer the method to as *pf-NMPC*. The *pf-NMPC* procedure includes the following three steps.

(1) Solve the NLP problem $\mathcal{P}_N (\boldsymbol{\chi}_{k+1})$ at time k constraining the initial state value to the next predicted state from the previous iteration.
(2) When the measurement $\mathbf{x}_{k+1}$ becomes available at time $k+1$, compute an approximation of the NLP solution $\mathcal{P}_N (\mathbf{x}_{k+1})$ using the QP (12) in a path-following manner.
(3) Implement the optimal control input and update $k \leftarrow k+1$ and repeat from Step 1.

Again, a sketch of the pf-NMPC procedure is described in Algorithm 1.

4. SIMULATION RESULTS

In this section we use a gas lifted well network with $n_w = 2$ wells to demonstrate the pf-NMPC controller and compare its results against the ideal NMPC (iNMPC) controller which solves the problem (3) using an NLP solver. All simulations are done in `MATLAB` using `CasADi` algorithmic differentiation tool (Andersson (2013)) version 3.2.0 where `IPOPT` is included as an NLP solver. We use the QP solver from TOMLAB MINOS (Murtagh and Saunders (1982)).

First, we run a steady-state optimization with total gas production capacity constraint equals to 9.5 kg/s. The solution obtained from this steady-state optimization is then used as setpoints. The optimized steady-state control inputs, algebraic and state variables are incorporated to the stage cost (4) as regularization terms, i.e.,

$$\begin{aligned}
\psi_m (\boldsymbol{\chi}_l, \boldsymbol{\nu}_l, \boldsymbol{\zeta}_l) = & \psi (\boldsymbol{\chi}_l, \boldsymbol{\nu}_l, \boldsymbol{\zeta}_l) \qquad (13)\\
& + \alpha (\|\boldsymbol{\chi}_l - \mathbf{x}_s\|, \|\boldsymbol{\nu}_l - \mathbf{u}_s\|, \|\boldsymbol{\zeta}_l - \mathbf{z}_s\|),
\end{aligned}$$

where $\mathbf{x}_s$, $\mathbf{u}_s$, and $\mathbf{z}_s$ are the steady-state state variable, control input, algebraic variable respectively.

Next, we run the NMPC controllers and initiate with an initial condition away from the optimal point. The NMPC controllers are implemented with sampling time of 5 minutes with a prediction horizon of 2 hours yielding 3014 optimization variables and 2966 nonlinear constraints in the NLP.

Algorithm 1 Economic pf-NMPC algorithm

Input: initial state $\mathbf{x}_0$, initial Δt, and η_{max}.

for $k = 0, 1, 2, \ldots$ **do**
  $[\boldsymbol{X}^*, \boldsymbol{\lambda}^*] \leftarrow$ solution NLP $\mathcal{P}_N\left(\boldsymbol{\zeta}_{k+1}\right)$ for $k+1$.
  if a measurement of $\mathbf{x}_{k+1}$ is available **then**
    Set $\boldsymbol{p}_0 = \boldsymbol{\zeta}_{k+1}$
    Set $\boldsymbol{p}_f = \mathbf{x}_{k+1}$
    $\boldsymbol{X} \leftarrow$ QP_PC_PF$(\boldsymbol{X}^*, \boldsymbol{\lambda}^*, \boldsymbol{p}_0, \boldsymbol{p}_f, \Delta t)$
    Inject the first input value from element of $\boldsymbol{X}$ to the plant
    Update initial state $\mathbf{x}_0 \leftarrow \mathbf{x}_{k+1}$
    Set $k+1 \leftarrow k$
  end if
end for

function QP_PC_PF$(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{p}_0, \boldsymbol{p}_f, \Delta t)$
  Define parameter γ satisfying $0 < \gamma < 1$.
  Define $\mathcal{A}_+$.
  Set parameter $\eta_{max} \ll 1$.
  Set $k \leftarrow 0$.
  Set $t_k = 0$.
  while $t_k < 1$ **do**
    Compute $\eta_k := \eta\left(\boldsymbol{X}_k, \boldsymbol{\lambda}_k, \boldsymbol{p}\left(t_k\right)\right)$.
    if QP is feasible **then** ▷ solve QP
      Compute $\eta_{k+\Delta} := \eta\left(\boldsymbol{X}_k + \Delta\boldsymbol{X},\ \Delta\boldsymbol{\lambda}, \boldsymbol{p}\left(t_{k+\Delta t}\right)\right)$.
      if $\eta_{k+\Delta} > \eta_{max}$ **then.**
        Decrease Δt. ▷ reduce QP stepsize
        $k \leftarrow k+1$.
      else
        $\boldsymbol{X} \leftarrow \boldsymbol{X} + \Delta\boldsymbol{X}$
        $\boldsymbol{\lambda} \leftarrow \Delta\boldsymbol{\lambda}$
        $t_{k+1} \leftarrow t_k + \Delta t$
        $\mathbf{p}\left(t_k\right) = \left(1 - t_k\right)\mathbf{p}_0 + t_k\mathbf{p}_f$
        if $\eta_{k+\Delta} < \eta_k^{1+\gamma}$ **then** ▷ very good step
          Increase Δt.
        end if
        Update $\mathcal{A}_+$. ▷ from QP's dual solution.
        $k \leftarrow k+1$.
      end if
    else
      Decrease Δt. ▷ reduce QP stepsize
      $k \leftarrow k+1$.
    end if
  end while
  return $\boldsymbol{X}$
end function

Output: The actual state $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, $\ldots$

We set $\eta_{max} = 10^{-5}$ and initial $\Delta t = 1.0$ for the pf-NMPC controller. Note that we use an adaptive steplength strategy for Δt in the Algorithm 1 meaning that the steplength Δt may be reduced in case $\eta_{max} > 10^{-5}$.

4.1 Comparison of Open-loop Optimization Results

We compare the open loop solutions from the ideal NMPC and pf-NMPC controllers at time t = 10 minute (at second NMPC iteration). The results are shown in Figure 2 in which the total oil and gas production are depicted. The solutions from pf-NMPC accurately track those of the ideal

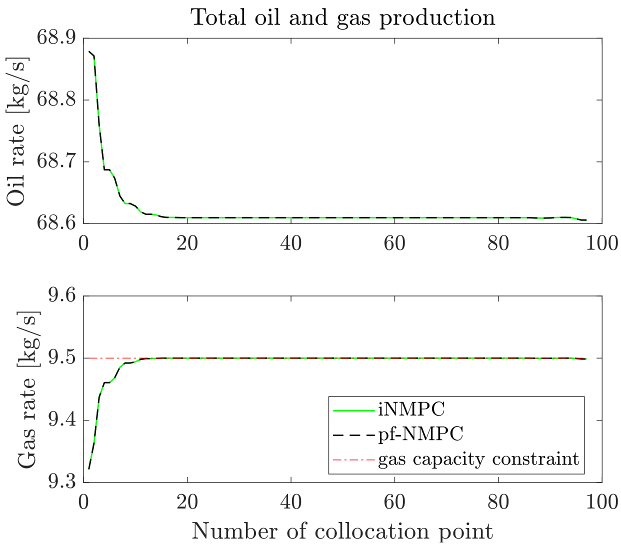

Fig. 2. Comparison of total oil and gas production for iNMPC and pf-NMPC controllers from open loop solutions at iteration number two. The solution of iNMPC is depicted in green color, which overlaps the solution of pf-NMPC denoted in black color.

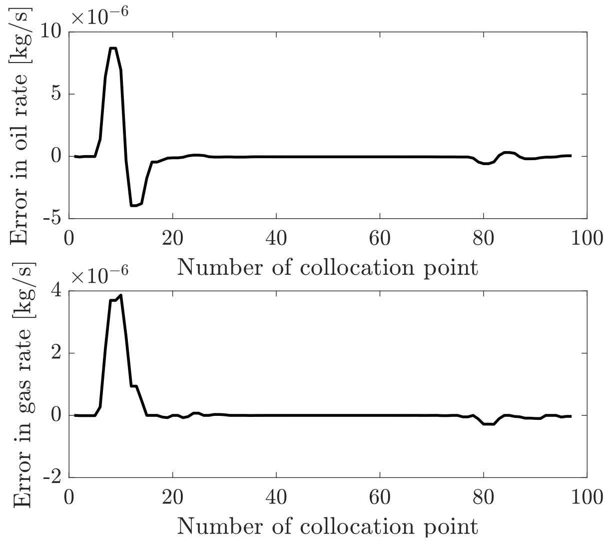

Fig. 3. Total production errors for open-loop solution at iteration number two.

NMPC controller. The errors between the ideal NMPC and pf-NMPC are plotted in Figure 3.

4.2 Closed-loop Results

Now, we check the closed-loop solution for both NMPC controllers. The solutions, control input profiles as well as total gas and oil productions, are displayed in Figure 4 and Figure 5. The total gas production capacity constraint, shown in Figure 5, is active. The solutions of pf-NMPC approximate the solutions of the ideal NMPC controllers quite nicely. Moreover, we compare the online optimization runtime for 60 NMPC iterations. It is shown that the pf-NMPC controller is able to speed up the optimization more than two times faster than those of the ideal NMPC controller.

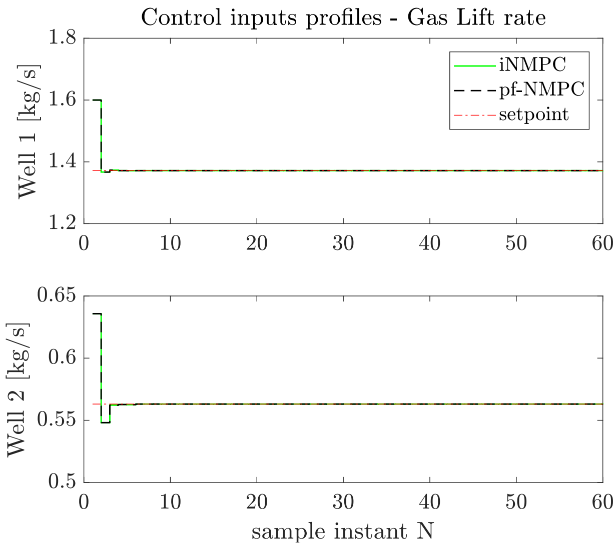

Fig. 4. Control inputs comparison of iNMPC and pf-NMPC controllers from closed-loop solutions.

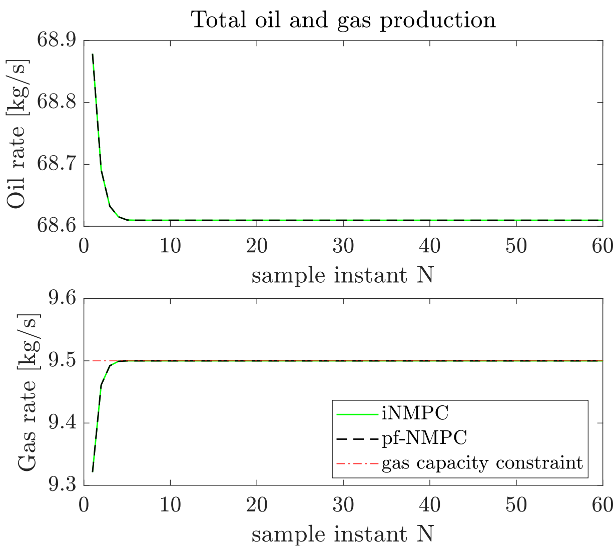

Fig. 5. Total production of oil and gas of iNMPC and pf-NMPC controllers from closed-loop solutions.

	online optimization runtime (in sec.)		
	min	max	average
iNmpc	0.85	0.94	0.88
pf-Nmpc	0.34	0.39	0.36

5. CONCLUSION

In this paper we presented an economic nonlinear model predictive control for a gas lifted well network. To address the issue of computational delay associated with economic NMPC, we presented a path-following predictor-corrector approach. Using simulation results, we showed that the pf-NMPC is able to provide fast solution while honoring the active constraints and reasonable approximate solutions.

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