Preprints of the 10th IFAC International Symposium on Dynamics and Control of Process Systems
The International Federation of Automatic Control
December 18-20, 2013. Mumbai, India

Dynamic online optimization of a house heating system in a fluctuating energy price scenario

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Abstract: We consider dynamic optimization of the energy consumption in a building with energy storage capabilities. The goal is to find optimal policies which minimize the cost of heating and respect operational constraints. The main complication in this problem is the time-varying nature of the main disturbances, which are the energy price and outdoor temperature. To find the optimal operable policies, we solve a moving horizon optimal control problem assuming known disturbances. Next, we proposed simple implementation based on feedback control, which gives a near-optimal operation for a range of disturbances. The methods were successfully tested in simulations, which show that there is a great economical gain in using dynamic optimization for the case of variable energy price.

1. INTRODUCTION

Recently, great attention has been given to renewable generation sources like wind turbine and photovoltaic parks. Although efficiency-wise attractive, these alternative energy sources suffer a major drawback due to their sharply varying energy production caused by wide-ranging weather conditions. This is an important limitation since the energy production should cover the demand at any given time.

One possible approach to overcome this drawback is demand side load management. Here, large fluctuations in the load are tackled by peak shaving and by shifting load to more beneficial periods (Molderink et al., 2009). This can be achieved by manipulating the energy price according to demand information and weather forecasts. The dynamic energy pricing for demand load management is in itself a non-trivial problem, and it is currently an active research area. The interested reader is invited to check the references Roozbehani et al. (2010) and Goudarzi et al. (2011) for more information. This problem is outside the scope of this work.

In such a scenario, the adaptation of the energy consumption by the final consumer is essential to the success of the approach. Thus, in this article we focus on the local building heating system optimization where the goal is the minimization of energy costs, which in turn will lead to lower consumption when energy is less abundant.

The case studied here consists of a single room comprised of a floor heating device, a radiator and a ventilation system with adjustable flow. We consider bounds on the floor temperature, the room temperature (air) and the CO_2 levels. The floor heat capacity is assumed to be large enough so that we can store a considerable amount of energy in it, hence, giving us an extra degree of freedom for optimization. Other hardware configurations could also have been employed. For example, one could use a insulated tank filled with water.

The main complicating factor for this problem is the time-varying nature of the disturbances in the outdoor temperature and energy price. We assume that predictions of the temperature and price variation are available, but they are not necessarily correct. Thus, a dynamic real-time optimization (DRTO) scheme is proposed to compensate this variations while minimizing the energy cost. In this scheme, a dynamic optimization problem is solved at each sample time with new states and disturbance measurements.

A drawback of the DRTO is the fact that the system operates in open-loop in between two consecutive optimizations. This may yield sub-optimal or even infeasible solutions in case of large disturbances. To deal with this problem, we propose simple solutions solely based on feedback and offline analysis, where near-optimal control inputs are generated at low computational and maintenance costs. This extends the self-optimizing control idea (Skogestad, 2000) to dynamic optimization problems. We show that near-optimal solutions can be obtained by tracking *optimally invariant trajectories*, which we define as a function of the measurements whose optimal profile does not change with disturbances.

The paper is organized as follows: Section 2 details the derivation of the model. Section 3 shows the formulation of the dynamic optimization problem and describes the solution method used. In Section 4, the implementation of the optimal solution is discussed. Section 5 gives the simulation results and Section 6 concludes the article.

2. MODELING

The model describes a single $25m^2$ room comprised of a floor heating device, a radiator and a ventilation system

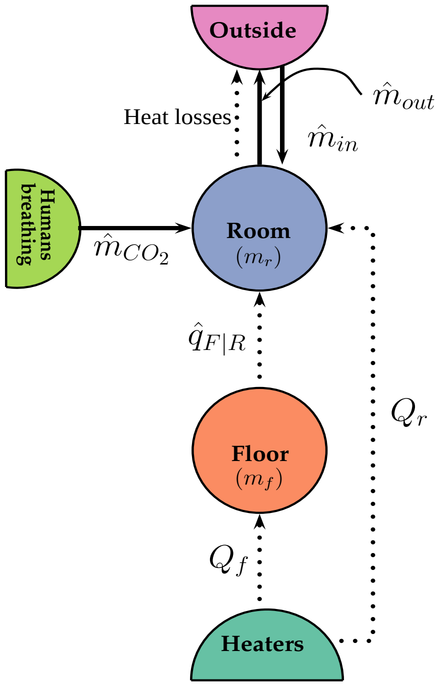

Fig. 1. The system topology

with adjustable flow. It is assumed that all the heat lost by the floor is transferred to the air in the room whereas the heat in the air can be lost both through the walls and through the ventilation. The air entering is assumed to be at outdoor temperature and behaves as an ideal gas. The CO_2 accumulation due to breathing is modelled as a constant feed B and the consumption of O_2 is neglected. To help visualizing the energy and mass flows in the system it is useful to use system topology graph as shown in Fig. 1. All state, manipulated and disturbance variables are described in Table 1. Other constant parameters are summarized in Table A.1.

Table 1. Variables description

State variables	Description	Unit
T_f	Floor temperature	K
T_r	Room temperature	K
m_r	Mass of air	kg
w	CO_2 mass fraction	-
Manipulated variables	**Description**	**Unit**
Q_f	Floor heat input	kW
Q_r	Room heat input	kW
$\hat{m}_{in}$	Air inflow	kg/s
Disturbance variables	**Description**	**Unit**
T_o	Outdoor temperature	K
p	Energy price	\$/kW
B	Rate of CO_2 generation	kg/s

From mass and energy balances, the dynamics of the system may be described as

$$\frac{dm_r}{dt} = \hat{m}_{in} - \hat{m}_{out} \tag{1}$$

$$\frac{dw}{dt} = \frac{\hat{m}_{in}}{m_r}(w_{in} - w) + \frac{B}{m_r} \tag{2}$$

$$\frac{dT_f}{dt} = \frac{Q_f}{m_f c_{p,f}} - \frac{UA_{f,r}}{m_f c_{p,f}}(T_f - T_r) \tag{3}$$

$$\frac{dT_r}{dt} = \frac{Q_r}{m_r c_{p,r}} + \frac{\hat{m}_{in}}{m_r}(T_o - T_r) + \frac{UA_{fr}}{m_r c_{p,r}}(T_f - T_r) + \frac{UA_{r,o}}{m_r c_{p,r}}(T_o - T_r) \tag{4}$$

where $\hat{m}_{out} = k(P_r - P_o)$ is the outflow and $P_r = \frac{m_r R T_r}{M_r V_r}$ is the pressure inside the room, where we assume ideal gas. The heat generated by humans and the effect of the sun light have been neglected. For sake of simplicity in the notation, we define the control inputs $u^T = [Q_f, Q_r, \hat{m}_{in}]$, the state vector $x^T = [T_f, T_r, m_r, w]$ and the disturbances $d^T = [T_o, p, B]$. Hence, we can pack the dynamics into the vector function f such that $\frac{dx}{dt} = f(x, u, d)$. In the next section we describe how to use this model to find optimal heating polices.

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3. DYNAMIC OPTIMIZATION

This section presents the dynamic optimization problem and the approach used to solve it. It starts off by presenting the continuous time optimal control problem we would like to solve and evolves in a stepwise manner presenting modifications that helps the solution. Finally, we present the full discretization method based on orthogonal collocation as well as the formulation of the nonlinear program. The implementation is discussed in the subsequent section.

3.1 Disturbance modelling

The main disturbances are the outdoor temperature $T_o(t)$ and energy price $p(t)$. For simplicity, we assume that $p(t)$ is periodic and follows

$$p(t) = p_0 + A_p \text{sign}[\sin(\omega_p t + \phi_p)] \tag{5}$$

where parameters the A_p and ϕ_p are uncertain. More general dynamic pricing polices can also be treated in this framework in a straightforward manner. We assume the weather predictions are available numerically from weather models such that we can interpolate the predictions using polynomials. Therefore, we assume we have the predictions $\hat{T}_o(t) = P(t)$ where P is a polynomial fitted using the weather model data. For this case study we have used weather prediction data from (yr.no, 2012). It would not be realistic to embed a weather forecast model in the optimization loop due to its highly complex nature. The rate of CO_2 generation by breathing (B) is a function of the number of people currently inside the building.

3.2 Dynamic optimization problem

The optimization objective is to minimize the energy costs over an infinite horizon. A solution method is to use a moving horizon approach where we solve an optimal control problem within the fixed interval $[t_0, t_0 + h]$ where the horizon h is large enough to capture important trends in the system. At each time point t_0 a different optimization problem (6) is solved with different initial condition x_0 that is unknown in advance. We formulate our moving horizon problem in the Lagrangian form as:

$$\min_u \int_{t_0}^{t_0+h} p(t)[\beta(Q_f+Q_r)^2+(Q_f+Q_r)]\,dt \tag{6}$$

subject to

$$\dot{x} = f(x,u,d), \quad x(t_0)=x_0 \tag{7}$$
$$T_r \geq T_{min} \tag{8}$$
$$T_f \leq T_{max} \tag{9}$$
$$w \leq w_{max} \tag{10}$$
$$Q_f \leq Q_{max} \tag{11}$$
$$Q_r \leq Q_{max} \tag{12}$$
$$Q_f, Q_r \geq 0 \tag{13}$$

where the quadratic term was introduced to improve numerical convergence of the optimization algorithm. The weighting factor β should be adjusted such that the linear term dominates the expression.

During operation is possible that a disturbance brings the system outside the feasible region. The formulation based on hard constrains (8)-(13) would then fail to produce a reasonable solution since the initial state would already be infeasible. This problem can be overcome by softening the output constraints (8)-(10). It would not make sense to soften the input constrains as they represent actual physical limitations.

Firstly, we rewrite the output constraints in a vector form such that we have $h_o(x,u) \geq 0$. Next, we introduce a vector of slack variables ε and define the following constraints in the optimization problem:

$$h_o(x,u) \geq 0 - \varepsilon \tag{14}$$
$$\varepsilon \geq 0 \tag{15}$$

Finally, the cost function is modified by adding penalties for the violation of the constraints

$$\min_u \int_{t_0}^{t_0+h} \{p(t)[\beta(Q_f+Q_r)^2+(Q_f+Q_r)]+\mu\cdot\varepsilon\}dt \tag{16}$$

The linear penalty function was chosen because it is exact in the sense that minimizing (16) also minimizes the original cost function (6) provided that μ is large enough (Nocedal and Wright, 2006).

3.3 Numerical solution: simultaneous approach

The optimal solution is obtained through the application of a simultaneous approach (Biegler, 2010), in which both the states and the inputs are approximated by orthogonal polynomials. For simplicity, we first transform the problem to the Mayer form by expanding the state vector with $\dot{J} = p(t)[\beta(Q_f+Q_r)^2+(Q_f+Q_r)]+\mu\cdot\varepsilon$ such that we have the augmented states $z^T = [x, J]$ and $\dot{z} = \hat{f}(z,u,d)$. The equivalent dynamic optimization problem is

$$\min_u J(t_0+h) \tag{17}$$

subject to the constraints (11)-(15) and the model $\dot{z} = \hat{f}(z,u,d)$.

Proceeding to the discretization, we first divide the time interval into N time periods. Within each time period i the control inputs are represented by Lagrange interpolation

$$u(t) = \sum_{j=1}^{K} \bar{l}_j(\tau)u_{ij} \tag{18}$$

where

$$\bar{l}_j(\tau) = \prod_{k=1,\neq j}^{K} \frac{\tau-\tau_k}{\tau_j-\tau_k} \tag{19}$$

The collocation equations for the differential equations can be written as

$$\sum_{j=0}^{K} \dot{l}_j(\tau_k)z_{ij} - h_i\hat{f}(u_{ik},z_{ik},d_{ik}) = 0 \tag{20}$$

where $i \in [1,\ldots,N]$, $k \in [1,\ldots,K]$, $\dot{l}_l(\tau) = \frac{dl_j}{d\tau}$ and K is the degree of the polynomials. The length of the time intervals h_i are considered fixed and are not decision variables for the optimization problem. In fact, for this case we have chosen $N = 1$ which leads to a *pseudospectral method*. Finally, the collocation points τ_k are chosen as the roots of the Gauss-Legendre orthogonal polynomials. The resulting NLP is as follows:

$$\min J(t_0+h) \tag{21}$$

$$\text{s.t.} \sum_{j=0}^{K} \dot{l}_j(\tau_k)z_j - h\hat{f}(u_k,z_k,d_k) = 0 \tag{22}$$

$$h_o(x_k,u_k,d_k) \geq -\varepsilon_k, \quad \varepsilon_k \geq 0, \quad h_u(u_k) \geq 0 \tag{23}$$
$$k \in [1,\ldots,K] \tag{24}$$

where $h_u(u) \geq 0$ represents the input constraints (11)-(13). The above problem is formulated in MATLAB and solved using the sparse NLP solver SNOPT. This solver employs a sparse SQP algorithm with quasi-Newton approximations to the Hessian. Gradient information is obtained using automatic differentiation approach. The interface between MATLAB and SNOPT is handled by the optimization environment TOMLAB.

4. IMPLEMENTATION OF THE OPTIMAL SOLUTION

4.1 Dynamic real-time optimization

We propose the implementation of a dynamic real time optimization where the optimal control problem is solved in a moving horizon fashion. At each time sample, t_0, a dynamic optimization problem is solved with a new initial state and disturbance measurements. We specified a horizon length h = 24h so that all the important dynamics are captured. However, only the first portion of the optimal profile corresponding to $t \in [t_0+t_s]$ is implemented, where $t_s < h$ is the time between successive optimizations. In this paper we assume limited computation power so that we need to have t_s = 2h. During this period the optimal inputs are extracted by using the Lagrange interpolation shown in (18).

In order to improve the accuracy of the solution and improve the convergence, the NLP is solved with successively larger number of collocation points, where the solution to the previous lower dimensioned problem is used as an initial guess for the next one. Here, we solve the NLP first with $K = 25$ and then using $K = 45$ collocation points. Another important point is the warm start of the NLP solver. This is done in two steps: first, the control inputs from previous solutions are shifted to to the next time window by assuming the inputs remain constant in the final time period. Then, the shifted inputs are used to simulate the model and the states are extracted. The shifted inputs and the simulated states are the initial guess to the next optimization problem.

4.2 Near-optimal solution by tracking optimally invariant trajectories

In this section, we propose a simple control implementation that gives near-optimal solutions without the need for re-optimization online. The main idea is to find a function of the measurements whose trajectory is optimally invariant to disturbances and then track the trajectory using standard feedback controllers. The control structure is shown in Fig. 5 where $c_r(t)$ is the optimally invariant reference trajectory that we wish to track. In the sequel, we will derive a procedure to obtain such trajectories.

We define $y \in \mathcal{R}^{n_y}$ as the vector of known variables (measurements), which may include states, disturbances and control inputs. The disturbance is modelled as a vector of constants d_0. However, the *real* (unknown) parameters are denoted by d, and we may have deviations $\Delta d = d - d_0$. The nominal optimal measurement trajectory is referred to as $y_0(t, d_0)$.

It can be shown that if the cost function J is twice continuously differentiable in a neighbourhood of the nominal solution and the linear independence constraint qualifications and the sufficient second-order conditions hold, then the optimal sensitivity matrix F is well defined:

$$F(t) = \frac{\partial y_{opt}(t, d)}{\partial d} \tag{25}$$

and, a first order, local approximation of the optimal solution in the neighbourhood can be obtained from

$$y_{opt}(t, d) \approx y_0(t, d_0) + F(t)\Delta d \tag{26}$$

We are searching for a function of measurements $c(y(t), d)$ whose optimal value is independent of d, that is, we want $c_{opt}(y(t), d) = c_0(y(t), d_0)$ for any d sufficiently small. A simple choice is a linear combination of the measurements:

$$c(t) \equiv H(t)y(t) \tag{27}$$

where $H(t)$ is a $n_u \times n_y$ matrix, and $c(t)$ is a $n_u \times 1$ vector. This way we can write

$$c_{opt}(t, d) = H(t)[y_0(t, d_0) + F(t)\Delta d] \tag{28}$$

and we define the nominal combination of measurements:

$$c_0(t, d_0) = H(t)y_0(t, d_0) \tag{29}$$

By subtracting (29) from (28) we obtain:

$$c_{opt}(t, d) - c_0(t, d_0) = H(t)F(t)\Delta d \tag{30}$$

Therefore, the optimal combination $c_{opt}(t, d)$ equals the nominal $c_0(t, d_0)$ for any d if we select $H(t)$ such that $H(t)F(t) = 0$. *This is always true if $H(t)$ lies in the left null space of $F(t)$.* Using this approach we obtain a trajectory $c_{opt}(t, d)$ that is optimally invariant to disturbances. We can transform the problem of implementing $u(t)$ in a 'open-loop' manner to a reference tracking problem with optimal setpoints $c_r(t, d) = c_{opt}(t, d)$ (see Fig. 5). By tracking c_r, a simple controller automatically generates inputs u that are optimal for any disturbance d sufficiently small and thus, the online optimization is avoided.

The whole procedure has offline and online steps which are summarized as follows:

Offline:

- Solve the dynamic optimization problem with d_0;
- Select appropriate measurements y;
- Compute the optimal sensitivities $F(t)$ and the combination $H(t)$;
- Compute the reference trajectories $c_r(t) = H(t)y_0(t)$.

Online:

- Track the reference c_r by a feedback controller.

Remark: It is only possible to choose H in the left null space of F if the number of independent measurements respect the condition $n_y \geq n_u + n_d$ where n_d and n_u are the number of disturbances and inputs, respectively. See (Alstad and Skogestad, 2007) for proof.

5. RESULTS

5.1 Nominal optimal solution

The solution for a whole day obtained with the DRTO algorithm was computed assuming perfect predictions. Figure 2 depicts the nominal price variations and the outdoor temperature variation. This temperature profile corresponds to the temperature measured in Trondheim, Norway on 03 January 2012 provided by the Norwegian Meteorological Institute which made the data freely available in (yr.no, 2012). For simplicity, we assumed a constant rate of CO_2 generation $B = 9.02 \times 10^{-6}\ kg/s$ in the simulations.

For the sake of comparison, we also implemented the most trivial solution to the problem where the room temperature is kept at minimum allowed value by varying the heat input Q_r using a PI controller. To get a fair comparison, the optimal air inflow was used. The second heat input,Q_f, was left unused. Note that keeping the room temperature at minimum allowed value is, in fact, the optimal policy if we would like to minimize the energy consumption instead of the economical cost.

A comparison between the optimal profiles and the simple strategy is given in Figures 3 and 4. Some interesting conclusions can be drawn from these results. First, notice that it is optimal to overheat the room and floor above the minimum constraint when the price is low. In this case, when the energy is cheap we will store enough heat in order to meet the temperature constraints until the next low price valley. We also confirmed (not shown here for brevity) that the air inflow is increased just enough to meet the CO_2 level constraint. This is trivial since over-ventilation would unnecessarily cool the room down and it would require extra energy to keep the temperature constraint.

The optimal energy cost for one day was \$12.45, whereas the simple temperature controller gave a cost of \$21.62, which is considerably higher than the optimal. The energy usage is 12.5kWh and 10.9kWh, respectively. This difference in the cost is proportional to the ratio between high and low energy price.

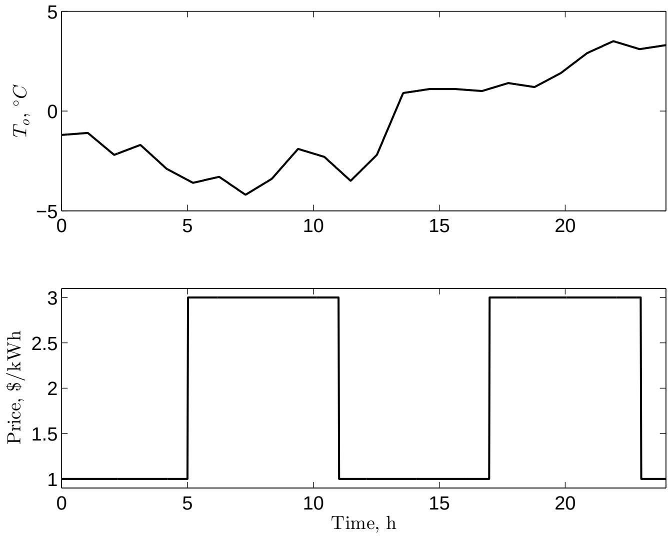

Fig. 2. Disturbances - energy price and outdoor temperature.

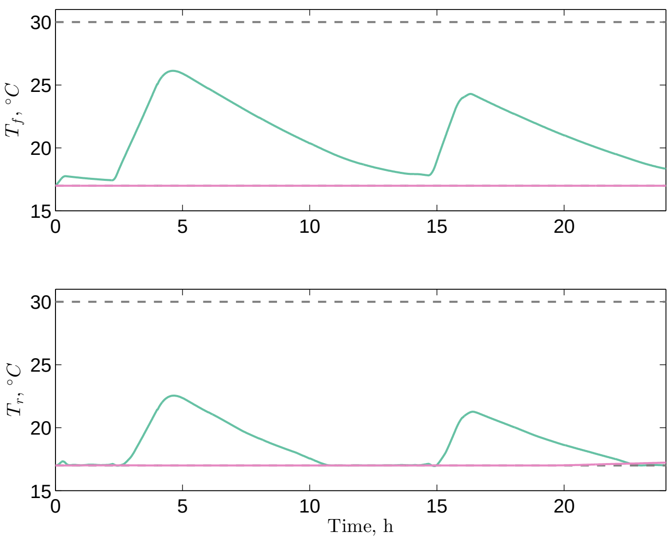

Fig. 3. Temperatures - green lines: optimal solution; magenta lines: simple temperature controller with constant setpoint.

5.2 Controlling invariant trajectories

Here, we assume the air inflow q_{in} will remain at nominal trajectory such that two manipulated variables are available. Thus, since we are considering two disturbances we will need at least $n_y = 2 + 2 = 4$ measurements and we seek two trajectories $c_1(t)$ and $c_2(t)$ to track. Defining the measurement vector $y = [T_f, T_r, m_r, p]^T$ we compute the optimal sensitivities $F(t)$ for the whole horizon and obtain $H(t)$ and the reference trajectory $c_r(t)$. As controllers, we use two decentralized P controllers. Note that the only way to adapt to price changes is by measuring it as the model of the physical process does not depend on price explicitly.

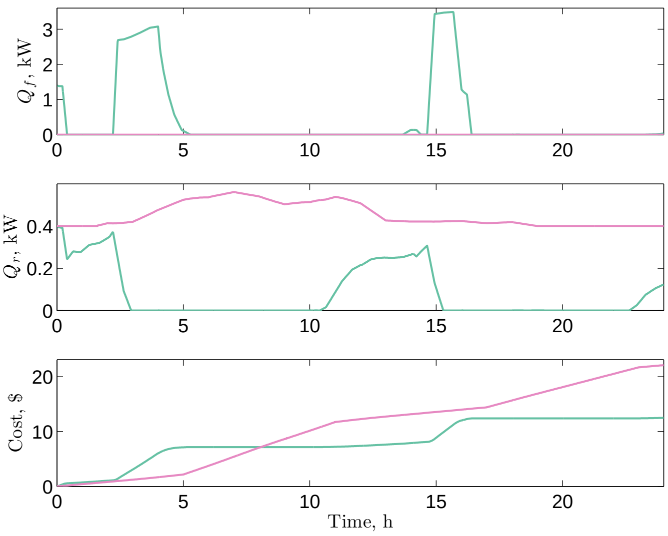

Fig. 4. Inputs and energy cost - green lines: optimal solution; magenta lines: simple temperature controller with constant setpoint.

This idea was tested by considering a disturbance in the phase shift (ϕ_p) of the energy price as well as a mismatch between prediction and actual outdoor temperatures. Figure 6 compares the predictions with the measured disturbance values. We compare the proposed method with the moving horizon strategy and with the true optimal solution assuming perfect knowledge of the disturbances. Figure 7 depict the input trajectories for the three different cases. The economical comparison is shown in bottom Fig. 7. The proposed simple method works very well for this case, given a relative loss of optimality of only 0.32%. The relative loss given by the moving horizon strategy with imperfect disturbance model was 24.4%, which is considerably higher.

5.3 Discussion

One of the reasons for the success of the method is the fact that, in this range of disturbances, the dynamics are close to linear and, therefore, the linear approximation of the NLP ends up near the true solution. A drawback of this approach is that it cannot explicitly handle constraints. Therefore, for a realistic implementation the proposed method should be combined with a periodic solution of the dynamic optimization where a new reference solution is obtained, and new invariant trajectories $c(t)$ are computed. The idea is to recompute the optimal sensitivities $F(t)$ online after solving the current NLP and then apply the approach shown in Fig 5 in between two successive optimizations. This requires, however, fast online calculations of the sensitivities as those provided by the methods proposed by (Pirnay et al., 2012). Similar idea has been published in (Würth et al., 2009) where the authors proposed to use sensitivity based neighbouring-extremal

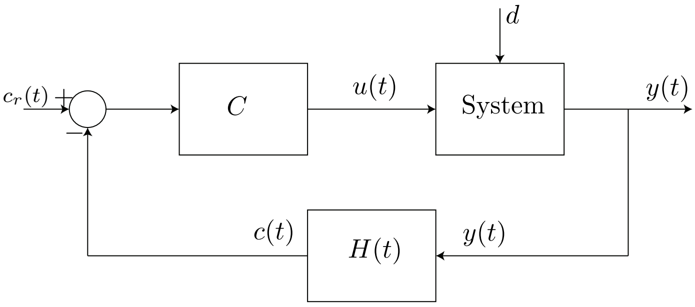

Fig. 5. Proposed implementation based on simple feedback

updates combined with real-time optimization. In this way, the frequency of optimizations can be greatly reduced.

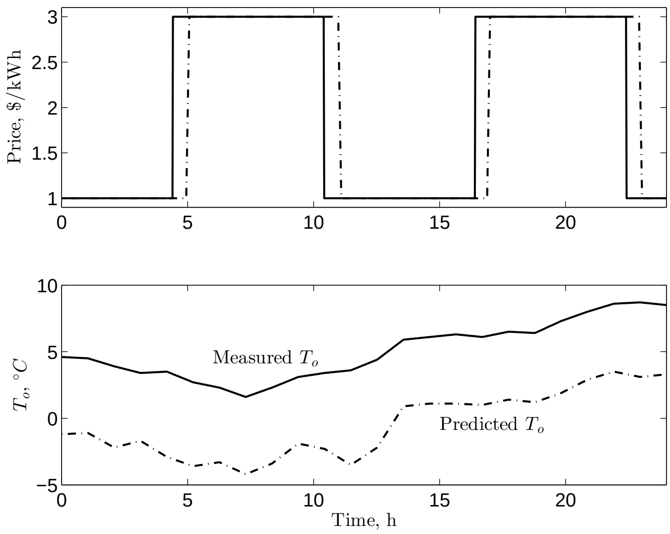

Fig. 6. Disturbances - solid lines: measured; dash-dotted lines: predicted

6. CONCLUSION

We proposed a moving horizon dynamic optimization method, which uses predictions to compute the optimal heating polices and ensure feasibility. We showed that, in a scenario where the energy price is time-varying, the economical benefit of using a real time dynamic optimization scheme is substantial. Finally, simple solutions based on feedback control and offline analysis was derived and successfully tested. The simulation example showed that very little loss of optimality could be obtained. The benefit of this method is the negligible online computational cost and the simplicity of the implementation. The ideas discussed here may also be applied to other problems with energy storage capabilities where the energy price changes.

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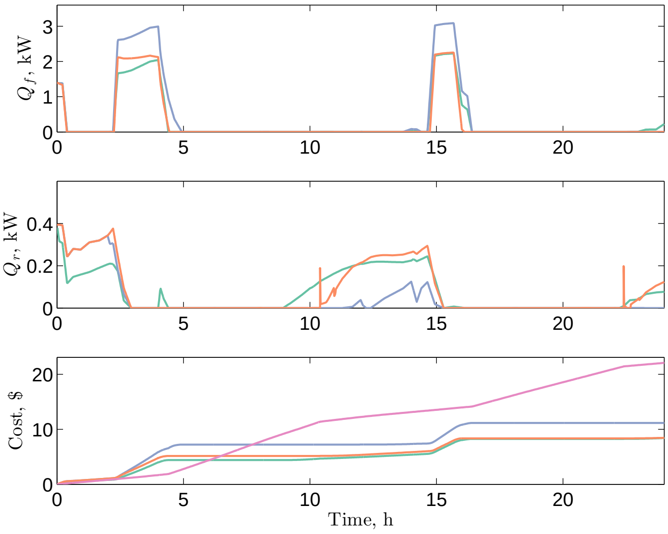

Fig. 7. Inputs and economical comparison - blue lines: DRTO with imperfect predictions; orange lines: proposed implementation as shown in Fig. 5; green lines: optimal solution; magenta line: cost of a simple temperature tracking controller

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Appendix A. MODEL PARAMETERS

Table A.1. Parameters description

Parameter	Description	Value	Unit
$UA_{f,r}$	Heat transfer coef. floor	0.1801	$kJ/(s \cdot K)$
$UA_{r,o}$	Heat transfer coef. walls	0.0216	$kJ/(s \cdot K)$
m_f	Mass of the floor	3000	Kg
$c_{p,f}$	Heat capacity floor	0.63	kg/kJ
$c_{p,r}$	Heat capacity air	1.005	kg/kJ
k	Valve constant	100	$kg/(bar \cdot s)$
w_{in}	CO_2 fraction in flow	$6.16 \cdot 10^{-4}$	-