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Health-Aware Control of a Subsea Compression System

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Preface

This thesis concludes the master's degree "Chemical Engineering and Biotechnology" at NTNU Trondheim, and was written at the Department of Chemical Engineering in the autumn of 2017. This thesis is done as part of the research project "Control for Extending Component Life" at SUBPRO, a Centre for Innovation-based Research (SFI) within subsea production and processing.

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I would like to thank my supervisors, Johannes Jäschke and Adriaen Verheylewegen, for the interesting and challenging topic and the opportunity to take part in an innovative field of research.

Declaration of Compliance

I declare that this is an independent work according to the exam regulations of the Norwegian University of Science and Technology.

Trondheim, February 16, 2018
Julie Marie Gjøby

Abstract

Subsea processing technology is enabling production from oil and gas fields with larger water depths, longer tie-back distances and harsher climate conditions. Although moving production and processing facilities to the seabed are creating new opportunities for the oil and gas industry, it is also arising new challenges. At the seabed the equipment is not easy accessible, and maintenance interventions are both time demanding and expensive. To reassure unanticipated breakdowns stringent requirements on safety and reliability are imposed on subsea processes, often leading to conservative designs and operation strategies. This may in turn lead to that the economic potential of the field is not fully realized.

A way to achieve better economical performance is to employ prognostics and health monitoring (PHM) in the decision making. Meaning that the state of the system is monitored and projected into the future, and considered when calculating the optimal control moves. A control structure that pro-actively adjusts the inputs to prevent a fault from occurring is often described by the term health-aware control. The aim of the current work is to explore the use of health-aware control on a subsea compressor subject to degradation.

A multi-layer control structure is developed and applied to a subsea compressor with the objective of maximizing the gas throughput while ensuring continuous operation until the next planned maintenance stop. To combine the short-term control objectives and the long-term profit and reliability targets the control structure is designed with a three level hierarchical architecture. In the top layer a dynamic real-time optimization (DRTO) problem is solved to find the long-term optimal operation strategy when the compressor is subject to load-induced degradation. For the given operation strategy, the optimal operating point is found in the below short-term optimizing layer. The computed set-points and parameters are taken into the supervisory control layer where self-optimizing control (SOC) is used to reject disturbances. In the regulatory control layer linear feedback control is used to stabilize the compressor.

The results from the current work show that the health-aware control structure are able to ensure operation of the compressor for the given time horizon. The health of the compressor are kept under the threshold value for the planned operating time, while maximizing the throughput.

Sammendrag

Undervannsprosesseringsteknologi muliggjør utvinning av olje og gass fra oljefelt på dypere vann, lengre tikoblingsavstander og i tøffere klima. Selv om flyttingen av prosesseringsenheter ned på havbunnen har skapt nye muligheter for olje og gass industrien, har den også gitt opphav til nye utfordringer. På havbunnen er ikke enhetene lettgjengelige for vedlikehold, og i tillegg er vedlikehold av slike undervannsenheter både svært dyr og tidkrevende. For å hindre uforutsette stopp i produksjonen blir strenge krav til sikkerhet og tilgjengelighet stilt. Dette leder ofte til konservative design valg og operasjonsstrategier, noe som igjen fører til at det økonomiske potensialet ikke blir utnyttet.

En måte å sikre bedre øknomisk utbytte er å benytte seg av tilstandsovervåkning og prognostikkverktøy under drift. Dette innebærer at tilstanden til enheten blir overvåket, predikert og regnet med når de optimale reguleringsbevegelsene bestemmes. En regulator som proaktivt endrer pådragene for å forhindre feil i å skje blir beskrevet som en tilstandsbevisstregulator. Målet med denne oppgaven er å undersøke bruken av tilstandsbevisstregulering av en undervannskompressor som brytes ned.

En regulatorstruktur bestående av flere regulatorsløyfer utvikles og påføres en undervannskompressor med det mål om å maksimere gasstrømmen samtidig som vedvarende produksjon fram til neste planlagte vedlikehold garanteres. Ved å kombinere de kortsiktige kontrollobjektivene med de langsiktige øknomiske og tilgjengelighetsmålene er strukturen bygd opp av tre lag, hver med en regulatorsløyfe eller et optimaliseringsproblem. I det øverste laget løses et dynamisk sanntids optimaliseringsproblem for å finne den langsiktige optimale produksjonsstrategien når kompressornedbrytingen er belastningsdrevet. For den resulterende produksjonsstrategien finnes det optimale operasjonspunktet og oppfylles i den underliggende overvåkende regulatorsløyfa. I tillegg brukes en enkel regulator for å stabilisere kompressoren.

Resultatene fra dette arbeidet viser at en tilstandsbevisstkontrolstruktur klarer å oppfylle kravene om vedvarende produksjon for den bestemte tidsperioden. Tilstanden til kompressoren er holdt under en gitt grenseverdi for hele produksjonstiden samtidig som gasstrømmen maksimeres.

List of Abbreviations

CCV	Close-coupled valve
CL	Closed loop
DRTO	Dynamic real-time optimization
FTC	Fault-tolerant control
HAC	Health-aware control
IPOPT	Interior-point optimizer
MPC	Model-predictive control
NLP	Non-linear program
OL	Open loop
PID	Proportional Integral Differential
PHM	Prognostics and health monitoring
RUL	Remaining useful life
SIMC	Skogestad Internal Model Control
SOC	Self-optimizing control

Contents

1

Introduction

To raise new questions, new possibilities, to regard old problems from a new angle, requires creative imagination and marks real advance in science.
Albert Einstein

In process systems engineering an aspiration is to utilize knowledge of the system behavior to achieve optimal overall performance. To achieve optimality, we must search for the *operational strategy* that will fulfill the *objective* defined for the system. The objective of a process reflects the desired outcome - most often being an economic aspect to maximize or minimize. No system, however well operated, can run without wear and tear on the equipment. Thus, a maintenance strategy is made for the system, in addition to the *process control and optimization* strategies, to ensure continuous operation. A multitude of maintenance policies can be applied to an industrial system, but they are mainly classified as corrective or preventive [Wang, 2002]. To prevent failure from occurring, the equipment is subject to periodically inspection or corrective action. The maintenance intervals are schedules based on knowledge of the potential system failures, and the policy may be based on system age or fixed time intervals. With the time between the maintenance intervals determined by statistical information, there is no guarantee that the system will not break down before the planned maintenance. To improve the system reliably, *Condition monitoring* techniques are employed together with *Prognostics and Health Monitoring* (PHM) techniques to improve the maintenance strategy. By predicting the *Remaining Useful Life*(RUL), maintenance actions can be scheduled when actually needed - cutting back unnecessary expenditures and system downtime. From being mainly utilized in predictive and reliability centered maintenance strategies, RUL estimations are now of growing interest for researchers to develop control and optimization strategies that pro-actively adjust for the system health.

The novel paradigm *Health-Aware Control* (HAC) combines the more established *Fault-*

tolerant control (FTC) with PHM to allow safe and reliable operation in the presence of failures, as well as adjusting the control actions to extend the components life. Condition monitoring and diagnostics are typically employed in FTC to provide acceptable dynamic performance subject to instrument faults and selected disturbances. There is a distinction between active and passive FTC. Where as the active strategies react to component failures by reconfiguration of the control actions, passive strategies aim at providing closed loop systems that are robust against uncertainties and restricted faults. Brown et al. [2009] introduces an improved FTC method implementing RUL estimations, but the goal is still to provide stable operation in the presence of failure and not to prevent failure from occurring. In recent years, a few authors have combined PHM on actuators and *model predictive control* to extend the RUL until maintenance action can be taken. The MPC principle is to combine feedback control with repeated optimization of the system model subject to constraints to obtain the optimal input trajectories Morari and Lee [1999]. PHM can be included in the constraints in the optimization or in the objective function itself. Pereira et al. [2010] included constraints on the accumulated actuator degradation in the optimization problem to distribute the control effort in a simulated tank level control system. Salazar et al. [2016] applied PHM in the constraints on pumps in drinking water network. Sanchez et al. [2015] introduced a fatigue-based prognosis in the objective function to minimize damage on wind turbine blades. HAC methods are not limited to MPC. As presented by [Escobet et al., 2012], HAC methods ensure safe, reliable and optimal operation until the next planned maintenance action by combining advanced control methods with:

- Data acquisition
- Condition monitoring and diagnostics
- Prognostics
- Decision-making

The main attributes of HAC is to combine deterministic system dynamics with stochastic system reliability to operate the process in the optimal manner.

When operation must be assured

One industry where the concept of HAC is of large interest is the subsea industry. Exhausting conventional oil and gas fields have pushed for industrial innovations to enable production from more remote and challenging fields. The development of subsea production and processing technology are enabling production from fields previously deemed infeasible, as well as prolonging production from exiting fields. Although, opening up for new business adventures, moving topsite installations down to the seabed have resulted in expensive and critical equipment being almost inaccessable for unplanned maintenance. Accessing the subsea equipment intales aquiering a special ship, ROV-s and expensive spare units. Not to mention sthe cost of system downtime and lost production. Thus - strict requirements are put on saftey and reliability during operation of such equipment. Especially subsea compressors can benefit from a control structure that optimize the trade-off between optimality and degradation. Rotating turbomachinery often have their most efficient operating point at the border to instability. Some authors have proposed the use of HAC on subsea compression system. In [Verheyleweghen and Jäschke, 2017a] diagnostics, prognostics and optimal operation is combined to ensure optimal economic profit without jeopardizing the plant reliability. The same authors, present in [Verheyleweghen and Jäschke, 2017b] a HAC strategy for combining reliabilty and operational considerations in an optimization problem with shrinking time horizon.

Thesis objective

The aim of the current work is to investigate how stable, reliable and optimal operation of a subsea compressor subject to degradation and shock damage can be ensured for a fixed time period. In order to achieve this, a hierarchical HAC structure is designed and applied to a compressor model. Since the main advocate for HAC is improved economic performance of the subsea compressor, the health-aware operation strategy is compared to the more conservative operation along a surge avoidance line. The HAC structure consist of three control layers, where the top level optimization layer extends the compressor RUL to hold the reminder of the time period and the two lower level control layers ensures optimal and stable operation of the compressor. On the top level, a *dynamic real-time optimization* (DRTO) problem is solved in discrete time to obtain the optimal stationary operating point fulfilling the operational constraints, by evaluating the current and predicted health of the system. The lower level control layers are separated from the top layer by time scale, providing a optimal and robust closed loop stationary model of the compression system along with health state indicators. A self-optimizing controller is employed in the regulatory control layer to remain optimality when the compressor is subject to small disturbances in the inlet condition and minor faults in the compressor performance. To ensure stable operation at high efficiency operating points, an active surge controller is implemented in the supervisory control layer.

The main focus is on the implementation and understanding of the concept of health-aware control, and not the development and implementation of models and methods for diagnostics and prognostics of subsea compressors. Thus, this thesis will not provide improved methods for condition monitoring, diagnostics and prognostics of subsea compressors. The aim is not a valid model, but rather a proof of concept.

Thesis structure

The theory of process control and optimization that is applicable to the current work is outlined in Chapter 2. Furthermore, an overview of the working principal of crompessors and the relevant theory of compression modelling are presented in Chapter 3. Next, the compressor models and the control structure is outlined in Chapter 4. The control loops are tested and the results are discussed in Chapter 5. Finally, concluding remarks and suggestions for further work is presented in Chapter 6.

2

Process control and optimization

Perhaps the central issue to be resolved by the new theories of chemical process control is the determination of control system structure.

Alan S. Foss

The goal of process control and optimization is to achieve stable and economic operation. Thus, a control system is employed. The actions of sensing, computing and actuation are the core of control systems. By knowing the behavior of a dynamic system, interactions make it possible to obtain the desired outcome. Eventhough control systems vary in complexity, they all build on the fundamentals of process control and optimization theory. This chapter sets out to provide the necassary theoretical background and the conseptual ideas used to design the health-aware control system explored in the current work.

Control structures

A control system is the combined structure of control loops designed to reach the operation objectives of a given system. As the complexity of the system increases so do the numbers of measurements and control loops - and thus the complexity of the control system. When designing the control system several structural decision must be taken. Morari et al. [1980] raised awareness on the gap between control of single units and large-scale integrated processes. To bridge this gap, Morari et. al, presented the concept of control structures. A control structure is made out of a set of variables to be controlled, a set of variables to be measured, a set of manipulated variables and a structure interconnecting these variables. Considering the general control configuration in Figure 2.1, control structure design deals with the structural decisions on selecting the variable sets of controlled outputs, y, manipulated variables, u, and measurements, y_m, selecting the control configuration - how u and y_m should be connected - and the selection of controller type.

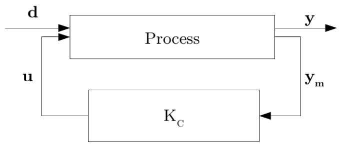

Figure 2.1: A general control configuration consist of a controller block K_C that takes the measurements y_m and computes inputs u to control a process subject to disturbances d.

The purpose of the control structure is to translate the economic objective into process control objectives [Morari et al., 1980]. In other words, the process is controlled to achieve best performance in terms of economics for a given set of operating conditions and constraints [Jäschke et al., 2017]. In order to achieve this, the control system is hierarchical structured into layers. The typical control structure often consist of a decision making layer, a supervisory control layer and a transient system with regulatory control - illustrated in Figure 2.2. The decision making layer and the control layers are separated by time scale, since scheduling and decision making are concerned with finding the optimal stationary operating point, whereas the control layers handle the dynamic regulation of the controlled variables.

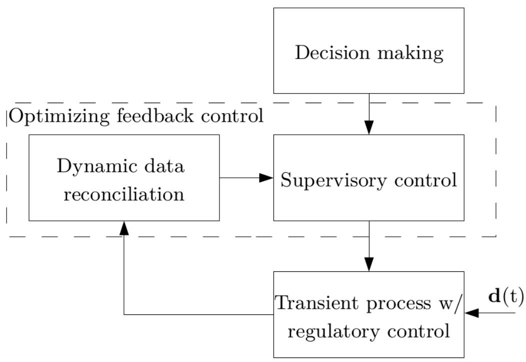

Figure 2.2: A control structure consist of a decision making layer, a supervisory controller and a regulatory controller. Here is the structure applied on a transient process with added dynamic data reconciliation. The process are subject to time varying disturbances ***d***.

Steady-state optimization

The goal of the decision making layer is to formulate and optimize production for the the operational objective. The operational objective is formulated as a *mathematical optimiza-*

tion problem with an *objective function*, *decision variables* and *constraints*. Normally, the objective function is a scalar function describing a economic property to be minimized. The decision variables can be selected from a *feasible set*, which is defined by the *equality constraints* and the *inequality constraints*.

$$\min_{u} \quad J(x, u) \tag{2.1}$$

$$\tag{2.2}$$

$$\text{subject to} \quad \boldsymbol{C_e}(x, u) = 0 \tag{2.3}$$

$$\boldsymbol{C_i}(x, u) \leq 0 \tag{2.4}$$

If the objective function is to maximize a property - for instance profit - the minimization can simply be altered to $\min_{x} -J(x, u)$.

Formulating the optimization problem is crucial - this involves identifying the decision variables, specifying the objective function and developing the process model expressed as equality and inequality constraints. First, process insights must be employed to identify the important independent (input $\boldsymbol{u}$) and dependent (output $\boldsymbol{y}$) variables. Second, the objective function must be specified in terms of the important process variables and operational goals. Typically, the objective function describe the performance of the process in an economic perspective. Finally, a *mathematical model* must be developed to describe the process by relating the input-output variables. In optimization applications, mathematical models based on physical and chemical laws - such as mass and energy balances, thermodynamics and chemical reaction kinetics, are frequently employed. When a physical model can not be developed, empirical models are a good alternative.

The solutions of Equation 2.4 is called *feasible points* or *minimizers*. For a solution to be optimal it must satisfy the *Karush-Kuhn-Tucker (KKT) conditions*. For more details on these conditions the reader is referred to [reference]. There may exit one, several or even no feasible points for a given optimization problem. For any given feasible point the active constraint is defined as a constraint for which $\boldsymbol{C_i}(x, z)$ = 0 - implying that all equality constraints are active.

Note that active constraints must be controlled.

Solving the optimization problem

Dynamic optimization optimize on a dynamic model - hence the solution will be a function of time. The system is sampled at discrete points in time with the sampling points being equidistant. Such a system is described by a finite difference model in the following state-space form:

$$x_{t+1} = g(x_t, u_t)$$

The control input, u_t, is assumed to be picewise constant and hence the change from u_t to u_{t+1} happens stepwise at $t + 1$. The state variables, x_t, are only defined at discrite points in time. The objective function of a dynamic optimization problem is defined on the *prediction horizon* spanned from $t = 0$ to $t = N$. It is formalized so that it sums the contribution from each time step:

$$f(x_N, u_N) = \sum_{t \in N} f_t(x_{t+1}, u_t)$$

Solving an optimization problem might be difficult, especially if it is non-linear and multivariate. Non-linear optimization problems are *non-convex problems* - meaning there might not be a optimal and feasible solution to the problem. In the current work an non-linear program (NLP) solver called the interior-point filter line-search (IPOPT) is used.

Process control

Control loops are typically *feedback control* loops. The working concept of feedback control is to measure the process output and adjust the control input until the output is at an acceptable value. Consider the transient process with a regulatory controller from the control structure in Figure 2.1. The system can be written on state-space form as:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) \tag{2.5}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{2.6}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{y}$ is the output vector, $\boldsymbol{u}$ is the input vetor, A is the state matrix, B is the input matrix and C is the output matrix.

A feedback controller will measure the error term, $\boldsymbol{e}$, as the deviation between the measure system output, $\boldsymbol{y}_m$ and the desired set-point, $\boldsymbol{y}_s$, and adjust the control input until the error is zero.

Controller design

A easy implementable and commonly used feedback controller is the *Proportional-integral-differential* (PID) controller. In time domain, the controller equation for a singel input-singel output control loop is written:

$$u(t) = u_0 + \left(K_c \cdot e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{d}{dt} e(t) \right) \tag{2.7}$$

The controller gains, K_C, K_I and K_D, are the controller parameters, and the response of the controller depends on the tuning of these. Severel tuning methods exists, one being the Skogestad Internal Model Control (SIMC). In practice the derivative part is often sat to zero Skogestad [2003]. The derivative action wants to counteract change in the system, thus working against the proportional and integral action in a way that destabilizes the controller. The advantages of the SIMC method are the facts that it is easy to use and that it results in a robust controller. By reducing the model or the response to a first-order plus time delay model in the frequency domain on the form:

$$y(s) = \frac{ke^{-\theta s}}{(\tau_1 s + 1)} u(s) \tag{2.8}$$

The process gain k, the time delay θ and the time constant τ_1 can be found by using the "half-rule" - the reader is reffered to [Skogestad, 2003] - or analytically as illustrated in Figure 2.3. A step is made in the input u and the resulting output is plotted.

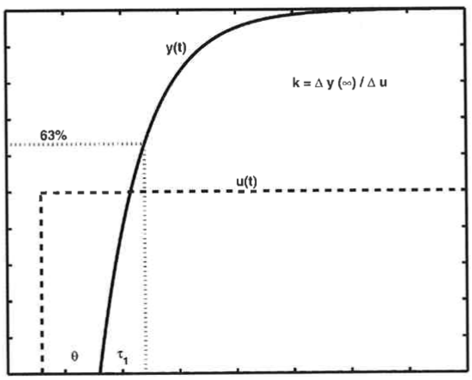

Figure 2.3: Identifying model parameters from step responseSkogestad [2003].

The recommended controller settings can be found from:

$$K_C = \frac{1}{k}\frac{\tau_1}{\theta + \tau_c} \tag{2.9}$$

$$K_I = \frac{K_C}{\tau_I} \tag{2.10}$$

$$\text{with} \tag{2.11}$$

$$\tau_I = min[\tau_1, 4(\theta + \tau_c)] \tag{2.12}$$

$$\tag{2.13}$$

The desired closed-loop time constant τ_C is typically sat equal to the time delay. This value gives a good trade-off between speed and robustness of the controller. Lower τ_C gives more aggressive controllers, whereas higher τ_C gives better robustness.

Self-optimizing control and the exact local method

The steady-state degrees of freedom remaining after the active constraints is controlled can be used to ensure close to optimal operation in the presence of disturbances. Such a control strategy, keep the deviation from the nominal solution as small as possible, is called *self-optimizing control* (SOC) [Skogestad, 2000]. Obviously, the best control variable would be the gradient of the objective function - if the gradient is zero, the process would be optimal. This variable, however, is unfortunately almost impossible to measure directly. The basic principle of SOC is to find a linear measurement combination, $\boldsymbol{c}$, to be kept at constant set points, $\boldsymbol{c}_s$ [Alstad and Skogestad, 2007].

$$\boldsymbol{c} = \boldsymbol{H} \cdot + \boldsymbol{y} \tag{2.14}$$

The selection matrix, H, is found such that the loss L, is kept at a minimum.

$$L = J(u, d) - J_{opt}(d) \tag{2.15}$$

A method to find the selection matrix is the exact local method. By considering the loss on the form. Given the loss:

$$L = \frac{1}{2}\boldsymbol{z}^T\boldsymbol{z} \tag{2.16}$$

with

$$\boldsymbol{z} = J_{uu}^{1/2} \cdot (u - u^{opt}) \tag{2.17}$$

Given a linearized model:

$$\Delta\boldsymbol{y} = G^y\Delta\boldsymbol{u} + G_d^y\Delta\boldsymbol{d} \tag{2.18}$$

$$\tag{2.19}$$

The magnitudes of the disturbances and the measurement errors as:

$$\Delta\boldsymbol{d} = \boldsymbol{W}_d\boldsymbol{d}' \tag{2.20}$$

$$\boldsymbol{n}^y = \boldsymbol{W}_{n^y}\boldsymbol{n}^{y\prime} \tag{2.21}$$

where W_d and W_{n^y} are diagonal scaling matrices with the magnitudes of the expected disturbances and measurement errors, respectively. $\mathbf{d}'$ and $n^{y\prime}$ are normalized vectors nominally distributed with zero mean and unity variance

The linearized version of the input is:

$$\Delta\boldsymbol{u}^{opt} = -J_{uu}^{-1}J_{ud}\Delta\boldsymbol{d} \tag{2.22}$$

With the sensitivity matrix:

$$\mathbf{F} = \frac{\partial\mathbf{y}^{\text{opt}}}{\partial\mathbf{d}} \tag{2.23}$$

Locally, using the truncated Taylor expansion yields:

$$\Delta\mathbf{y}^{\text{opt}} = \mathbf{F}\Delta\mathbf{d} \tag{2.24}$$

The sensitivity can be written as:

$$\mathbf{F} = (-\mathbf{G}^y\mathbf{J}_{\mathbf{uu}}^{-1}\mathbf{J}_{\mathbf{ud}} + \mathbf{G}_{\mathbf{d}}^y) \tag{2.25}$$

Using the linearizations, the loss can be written as:

$$\mathbf{z} = \mathbf{M}_d\mathbf{d}' + \mathbf{M}_{\mathbf{n}^y}\mathbf{n}^{\mathbf{y}\prime} \tag{2.26}$$

where

$$\mathbf{M}_d = -\mathbf{J}_{\mathbf{uu}}^{1/2} \left(\mathbf{H}\mathbf{G}^{\mathbf{y}}\right)^{-1} \mathbf{H}\mathbf{F}\mathbf{W}_{\mathbf{d}} = \mathbf{J}_{\mathbf{uu}}^{1/2} \left(\mathbf{J}_{\mathbf{uu}}^{-1}\mathbf{J}_{\mathbf{ud}} - \mathbf{G}^{-1}\mathbf{G}_{\mathbf{d}}\right) \mathbf{Wd} \tag{2.27}$$

$$\mathbf{M}_{\mathbf{n}^y} = -\mathbf{J}_{\mathbf{uu}}^{1/2} \left(\mathbf{H}\mathbf{G}^{\mathbf{y}}\right)^{-1} \mathbf{H}\mathbf{W}_{\mathbf{n}^{\mathbf{y}}} = \mathbf{J}_{\mathbf{uu}}^{1/2}\mathbf{G}^{-1}\mathbf{W}_{\mathbf{n}^{\mathbf{y}}} \tag{2.28}$$

The average loss satisfying eq.(2.27)-(2.28) is shown by [Kariwala et al., 2008] to be:

$$L_{\text{avg}} = \frac{1}{2}|| \left[\mathbf{M}_d \quad \mathbf{M}_{\mathbf{n}^y}\right] ||_F^2 \tag{2.29}$$

F is Forbenius norm. Average loss means the avareage between all possible distrubances nd implementation errors. The term to be normed can be written as:

$$\left[\mathbf{M}_d \quad \mathbf{M}_{\mathbf{n}^y}\right] = \mathbf{J}_{\mathbf{uu}}^{1/2} \left(\mathbf{H}\mathbf{G}^{\mathbf{y}}\right)^{-1} \mathbf{H}\mathbf{Y} \tag{2.30}$$

with:

$$\mathbf{Y} = \left[(\mathbf{G}^{\mathbf{y}}\mathbf{J}_{\mathbf{uu}}^{-1}\mathbf{J}_{\mathbf{ud}} - \mathbf{G}_{\mathbf{d}}^{\mathbf{y}})\mathbf{W}_{\mathbf{d}} \quad \mathbf{W}_{\mathbf{n}^y}\right] \tag{2.31}$$

The goal is to minimize the loss:

$$\min_{\mathbf{H}} = ||\mathbf{J}_{\mathbf{uu}}^{1/2} \left(\mathbf{H}\mathbf{G}^{\mathbf{y}}\right)^{-1} \mathbf{H}\mathbf{Y}||_F \tag{2.32}$$

The solution simplifies to:

$$\tilde{\mathbf{H}}^{\mathsf{T}} = (\mathbf{Y}\mathbf{Y}^{\mathsf{T}})^{-1}\mathbf{G}^{\mathbf{Y}} \tag{2.33}$$

3

Compression

The purpose of this chapter is to present the necessary theory used to model the compressor in the current work. A centrifugal compressor is studied in the current work, and thus will the working principal and behavior of centrifugal compressor be the focus in this chapter.

Compressors are used in a wide variety of industrial processed to increase the pressure of gases. The pressure increase in a centrifugal compressor is achieved by first accelerating the gas in a rotating impeller, and then converting the increased kinetic energy into pressure rise by decelerating the gas with the stationary diffuser. A centrifugal compressor is made out of a stationary casing containing the rotating impeller, the stationary diffuser and the volute.

Compressor performance

The steady-state performance of a compressor can be depicted graphically in a compressor map. A compressor map as the one depicted in Figure 3.1 is made out of speed lines and efficiency islands. Steady-state operation of the compressor is determined by the speed lines, also called compressor characteristics. For a given rotaional speed, the assosiated pressure rise is plotted for the flow. The efficiency curves are found from evaluating the efficiency for a given operating point. Compressor efficiency can be calculated by comparing the ideal compressor work with the actual work:

$$\eta = \frac{W_s}{W_a} \tag{3.1}$$

Compressor efficiency is defined as polytropic or isentropic depending on the PV^n relation. For $n = \gamma$, the compression is said to be a isentropic process.

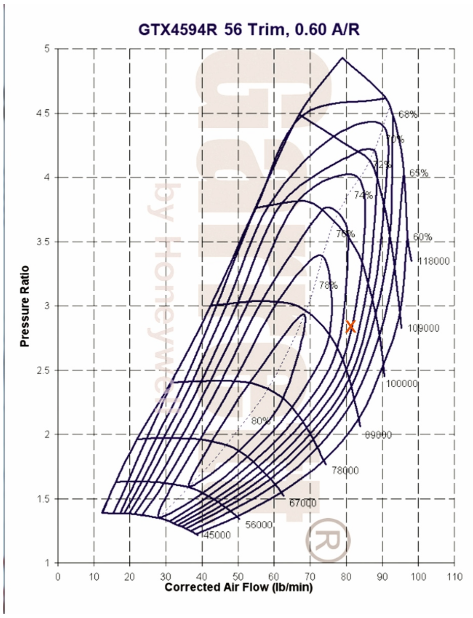

Figure 3.1: Illustrative compressor map with speed lines and efficiency islands. [Figure from powerperformancenews.com (2018)]

The resulting thermodynamic relation for the efficiency is a curve, but the efficiecies can be transformed into islands in the compressor map.

Disturbances

In the study of compressor performance, Lapina [1982] investigated how changes in inlet pressure, temperature, molecular weight, compressibility and specific-heat ratio would affect the compressor performance. Summed up, all changes can be represented by increasing or decreasing the parameters describing the characteristics - W, H and ψ_{c0}. Note that the changes were studied as isolated incidents, which in reality does not hold, since changes in these physical parameters are connected.

Instabilities

On the compressor characteristic curve there is a point which separates the compressors stable and unstable operating regions. This point is called the stall point or the surge point. In Figure this is the intersection between the black dotted line by the stalled region and one characteristic curve. The line connecting the stall points at different rotational speeds is called the stall line or surge line. Are compressors throttled past the stall line, the axisymmetric flow pattern becomes unstable. The instability can result in two phenomenon known as surge and rotating stall [Greitzer, 1981]. Surge is the phenomena of large amplitude oscillations of the total annulus averaged mass flow through the compressor. Rotating stall is the phenomena of cells of stalled flow rotating in the annulus.

In deep surge flow reversal occurs for part of the cycle. Surge affects the entire com-

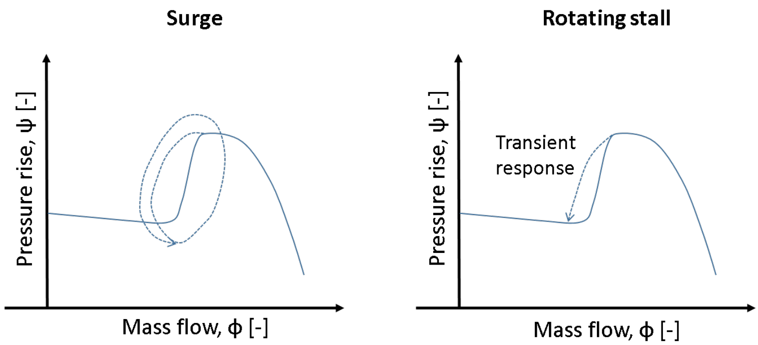

Figure 3.2: The transient response of the two stall phenomenon, surge and rotating stall.

pression system, and can cause severe structural damage on the equipment.

Dealing with the surge instability is important to ensure safe operation for the equipment and to achieve the most efficient operation possible. Effort is continuously made to develop and improve methods that deal with surge. The most common approach in the industry is surge avoidance [Yoon et al., 2013]. Surge avoidance methods are easy to implement, but trade-off the efficiency of the compression system. The principal of surge avoidance is to force the compressor to operate in a safe distance from the surge line. A flow controller will increase the flow and lower the pressure built-up when a operating point near the surge point is reached. The set points for the flow controller are generated parallel to the surge line, and the line connecting them is called the surge avoidance line. The distance between the surge avoidance line and the surge line is called the surge margin. Due to the uncertainty of the actual location of the surge line, large surge margins have to be enforced to avoid surge. Figure 3.3 illustrates the locations of these lines. From the figure it is clear that surge avoidance lowers the performance of the compression system by preventing operation in the high-pressure region of the compressor characteristic curve.

In the need for more efficient compressors, surge can be suppressed rather than avoided. By implementing a passive or active surge controller the surge induced disturbances are compensated for and the flow is stabilized. Whereas surge avoidance methods keep the compressor operating in a conservative region away from the surge line, surge control methods stabilize the compression system beyond the surge line and make more efficient compressors feasible. The passive surge controllers suppress the surge oscillations with passive elements that react to the environmental changes in the compressor. The passive elements can be said to be physical modifications designed to extend the stable operating region. Gysling et al. [1991] introduced the now most common passive surge control

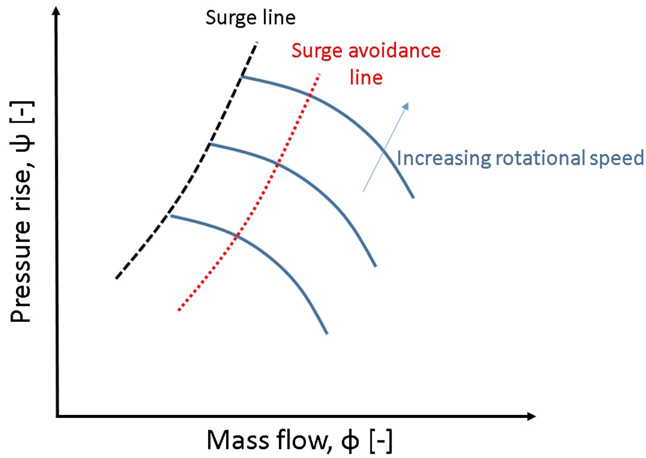

Figure 3.3: The compressor characteristic plot with an illustrative surge line and surge avoidance line.

strategy where a pressure variation in the compressor is induced by varying the size of the plenum volume with a mass-spring-damper system. This control method is further discussed in Arnulfi et al. [2000]. The active surge controller stabilize the surge oscillations with a sensor-actuator pair that actively perturb the compression system based on feedback measurements. Selecting the appropriate feedback sensor-actuator pair is key for making an effective active surge control strategy. Sensors are susceptible to noise, and can in worst case scenarios add destabilizing dynamics to the system [Yoon et al., 2013]. Actuators need to have enough bandwidth to stabilize the flow dynamics during surge, and actuator bandwidth may prove to be an important constraint in practical implementations [Simon et al., 1993]. Research and experiments have been done for several sensor-actuator pairs. Simon [1993] presented a surge controller for centrifugal compressors that uses a closed-coupled valve (CCV) as actuator. This controller is implemented in the compression system introduced in this project.

Compressor modelling

Modeling a compression system is a problem of classical fluid dynamics. The aim is to model the fluid properties as functions of time. In the presence of surge, the properties of interest are the flow velocity and the pressure. The preferred model for centrifugal compression systems is known as the Greitzer model [Yoon et al., 2013]. By applying the mass continuity equation and Newton's second law of motion to the control volume defined by a compression system, Greitzer [1976] developed a lumped-parameter model able to model the oscillations associated with surge and stall.

The mass continuity equation states that no mass can be created nor destroyed inside the

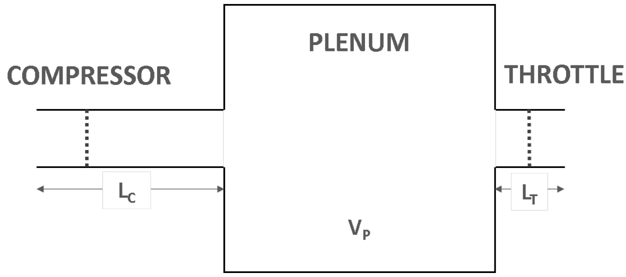

Figure 3.4: Schematic represetation of compression system used in the Greitzer model. The compressor is replaced by an actuator disk.

control volume. In other words, the net rate of change of mass inside a control volume must equal the net rate of flow into the volume. This is expressed as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \tag{3.2}$$

where ρ is the density, $\boldsymbol{v}$ is the velocity vector, t is time variable and ∇ is the vector differential operator.

Newtons second law states that the the net change of momentum for a control volume is proportional to the net force applied to the volume. For a inviscid fluid, the conservation of momentum equation is stated by the Euler equation:

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla (\rho \boldsymbol{v} \boldsymbol{v}) = -\nabla p + \rho \boldsymbol{g} \tag{3.3}$$

where p is pressure and $\boldsymbol{g}$ is the gravitational acceleration term.

The Greitzer Model

The compression system used to derive the Greitzer model is illustrated in Figure 3.4 . The system consist of a compressor with ducting working in series between a large reservoir with constant ambient pressure and an exit plenum volume, from which the discharge is through a throttle in an exit duct [Gravdahl and Egeland, 1999]. Oscillations occurring in such a system is modeled as those of an Helmholtz resonator - implying that all the kinetic energy of the oscillations are associated with the fluid motion in the compressor and the duct, and the potential energy is associated with the compression of the gas in the plenum.

When applying the conservation laws, the compression system is divided into three separate control volumes - the inlet duct, the plenum volume and the exit duct. The flow in the ducts are considered to be incompressible, in opposite to the gas in the plenum which is compressible. Assuming incompressible flow with the density of the ambient value in the ducts holds as the flows considered are those having low inlet Mach numbers and small pressure rises compared to the ambient, in addition to the fact that the oscillations associated with surge are regarded as having quite low frequency. Replacing the compressor and it's duct, as well as the throttle, with actuator discs and equivalent ducts (a length of constant area pipe) allows for the conservation laws to be applied in a one-dimensional manner along the pipe length. With an actuator disc is meant a rotor modeled as an infinitely thin discs with constant velocity along the axis of rotation. The simplification of the dimensionality of the conservation laws is accounted for by the assumption that the axial flow will behave lumped due to the incompressibility and the geometry of the equivalent ducts given as:

$$\left(\frac{L}{A}\right)_{eq} = \oint_S \left(\frac{dS}{A(S)}\right)_{actual} \tag{3.4}$$

where L is the effective length of the equivalent duct, A is the area and S is the streamwise direction.

The continuity equation is entered implicitly in the definition of the equivalent lengths, such that the rate of change of mass flow in the ducts are given as:

$$\frac{d\dot{m}_C}{dt} = \frac{A_C}{L_C}\left(C - \Delta P\right) \tag{3.5}$$

$$\frac{d\dot{m}_T}{dt} = \frac{A_T}{L_T}\left(\Delta P - F\right) \tag{3.6}$$

where subscripts C and T indicated the compressor and throttle duct, respectively, $\dot{m}$ is the mass flow, C is the pressure rise across the compressor, F is the pressure drop across the throttle and ΔP is defined as the pressure difference across the compressor duct.

Fluid velocities are negligible in the plenum, and the pressure is assumed to be uniform throughout the plenum. The continuity equation for the plenum yield:

$$\frac{d\rho_P}{dt} = \frac{1}{V_P}\left(\dot{m}_C - \dot{m}_T\right) \tag{3.7}$$

where V_P is the plenum volume.

The compression in the plenum is assumed to be polytropic, with a polytropic coefficient equal to the specific heat ratio γ so that:

$$\frac{d\rho_P}{dt} = \frac{\rho}{\gamma P_P}\frac{dP_P}{dt} \tag{3.8}$$

Since the overall pressure and temperature ratios studied are near unity, the mass conservation for the plenum may be written:

$$\frac{dP_P}{dt} = \frac{\gamma P}{\rho V_P}\left(\dot{m}_C - \dot{m}_T\right) \tag{3.9}$$

The Greitzer model is often used in an dimensionless and simplified version. The mass flows are nondimensionalized by using the factor $(\rho U A_C)$, the pressures using $(\frac{1}{2}\rho U^2)$ and the time by using the characteristic time, $(\frac{1}{\omega_H})$. Here ρ is the density, U the mean rotor velocity, A_C the compressor flow area and ω_H the Helmholtz frequency. The Helmholtz frequency is given by:

$$\omega_H = a_s\sqrt{\frac{A_C}{V_P L_C}}$$

where a_s is the speed of sound,.

By neglecting the inertial forces in the throttle duct, the Greitzer model is simplified to:

$$\dot{\phi} = B[\Psi_C(\phi) - \psi] \tag{3.10}$$

$$\dot{\psi} = \frac{1}{B}[\phi - \Phi_T(\psi)] \tag{3.11}$$

where B is the Greitzer B-parameter, the non-dimensional compressor mass flow is represented by ϕ, the non-dimensional compressor pressure rise is represented by Ψ_C, the non-dimensional pressure difference across the duct is represented by ψ, the non-dimensional throttle mass flow is represented by Φ_T. The Greitzer parameter is given by:

$$B = \frac{U}{2a_s}\sqrt{\frac{V_P}{A_C L_C}} \tag{3.12}$$

The Greitzer lumped parameter model adequately describes the oscillation associated with surge, but the model has its shortcomings. Yoon et al. [2013] highlight that the strength of the Greitzer model is also the models greatest weakness. The lumping of distributed flow parameters limits the geometry and dimensions of the the components in the compression model that the Greitzer model can describe. Research has shown that extended models must be used to model compression systems with exhaust piping. van Helvoirt and de Jager [2007] made enhancements to the Greitzer model in order to model the effect of the pipeline. The same model was used by Yoon et al. [2013]. The Greitzer model does not account for varying rotational speed. Fink et al. [1992] enhanced the model by including simple rotor dynamics. A model for centrifugal compressors with non-constant rotational speed was presented by Gravdahl and Egeland [1999] based on the Greitzer model. Note that the Greitzer model was derived for axial flow compressors, but it also holds for centrifugal compressors [Hansen et al., 1981].

Compressor characteristics

In the Greitzer model Ψ_C is the axisymmetric characteristic for a compressor without stall instabilities. Ψ_C is a hypothetical, and is argued to be a smooth S-shaped curve [Koff, 1983]. It is important to remark that the axisymmetric peak point is assumed to be the surge point for the given compressor. A simple cubic, as the one shown in Figure 3.5, is used to represent the calculated axisymmetric characteristic [Moore and Greitzer, 1986].

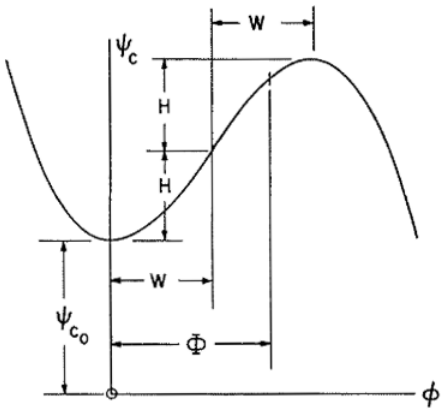

Figure 3.5: Cubic axisymmetric compressor characteristic with parameters indicated [Moore and Greitzer, 1986].

4

Model development

The sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is meant a methematical construct which, with the addition of certain verbal interpretations, describe observed phenonema. The justification of such a mathematical construct is solely and precisely that it is expected to work - that is, correctly to describe phenonema from a reasonable wide area.

John von Neumann

The purpose of the present chapter is to present the health-aware control (HAC) structure and the compression system being studied, focusing on their equation structures and underlying assumptions. Before any equations are specified, the control structure and the function of each control layer are outlined. A process description is given before the system model is explained. Finally, the control layers are designed.

As stated in Section 1.2, the ambition is not to develop a model that realistically represent any real industrial compression system. The ambition is rather to use a pure theoretical correlation to shed light on the effects of monitoring the system health in the optimization strategy. The main aim is to illustrate the potential of a health-aware control.

Overview

The underlying process investigated in the current work is subsea compression of natural gas. The natural gas is assumed to have constant composition and to be governed by the ideal gas equation of state

$$P = \frac{NRT}{V} \tag{4.1}$$

where P is pressure, T is the absolute temperature, N is the total mole number and R is the universal gas constant.

Operation of the subsea compressor is studied for a time period of five years, as this is the scheduled maintenance interval. During this time period, natural gas is extracted from a reservoir with declining pressure and a nominal pressure of 50 bar. The gas density ρ will change according to Equation 4.1 with:

$$\rho = \frac{M}{V} = \frac{N \cdot M_M}{V} \tag{4.2}$$

where M is the total mass and M_M is the molecular weight.

The gas is assumed to have constant composition and a molecular weight of 19 g/mole. After the compression, the outlet pressure must be at least 100 bar. To meet this pressure constraint, a centrifugal compressor with variable speed drive is used. The rotational speed ranges from 120000 to 160000 RPM and the maximum efficiency is 90 %.

The control structure

A HAC structure is designed and applied to the subsea compressor in order to ensure continuous and optimal operation until the next planned maintenance action. The HAC structure is hierarchical structured into a health aware optimization layer, a supervisory control layer and a regulatory control layer - ensuring, respectively, stable, optimal and reliable operation. Note that there is an important distinction in the frameworks in which the layers operate. In the optimization layer, time is viewed as an *discrete* variable, while in the control layers time is viewed to be *continuous*.

As mentioned in Chapter 1, HAC consist of the four modules: data acquisition, condition monitoring and diagnosis, prognostics and decision-making. In the proposed control structure, the health-aware optimization layer handles the decision-making for operation of the subsea compressor based on the current state of the system and realization of an economic goal and the predicted system degradation. Aligned with Chapter 3, the compressor behavior is defined by how fast the compressor blade is run and where on the speed line the compressor operates. Thus, these are the degrees of freedom in the optimization- from now on called the compressor settings. The control layers are not employed to contribute in the decision-making, but rather to achieve the compressor settings realizing the operational objective when the compressor is subject to disturbances. A disturbance often result in sub-optimal operation, and to keep the operation close to optimal in the presence of disturbances the supervisory control layer is employed. By using the set-point in the regulatory control layer as a degree of freedom, the regulatory control layer keep the loss, as defined in Equation 2.26, as small as possible. The regulatory control layer stabilizes the process to reach the set-point provided by the supervisory control layer. Due to the distinction in the time variable, the closed-loop compression system works as the steady-state system

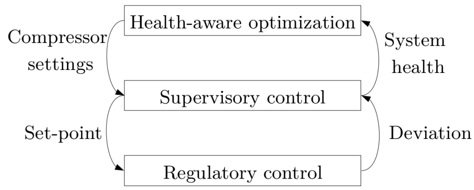

Figure 4.1: The control structure is hierarchical structured with a health-aware control optimization layer, a supervisory control layer and regulatory control layer. The flow of information between the layers are depicted.

model in the optimization layer. It is in the steady-state model that important variables are measured, building the foundation for the diagnostics and prognostics models employed to assess the system health during operation. Figure 4.1 illustrate the information flow within the HAC control structure.

Assumptions

For the overall model implemented in this work, the assumption of *perfect measurements* is invoked. It is also assumed that all parameters are *measurable*.

The gas reservoir is assumed to *lumped* and *well-mixed* , meaning that there is no spatial variation in the state variables. For the given time period, the reservoir temperature is assumed to be constant. As mentioned earlier, the natural gas is assumed to have constant composition. An assumption is made considering the validity of describing the compressor behavior in dimensionless time. Some have argued that the dimensionless Greitzer model is only valid when surge is present. For this work, it is assumed that the behavior described by the Greitzer model represent the behavior of the compressor sufficiently. The oscillations in mass flow and pressure in the dimensionless time domain are assumed to be representative for the degradation in real life. The Greitzer model are valid for compressor with small pressure rises compared to the ambient pressure. In this work it is assumed that the model equation are also valid for a compressor supplying a large pressure rise compared to the ambient pressure.

A cubic representation of the compressor characteristics are choosen to map the investigated compressor. The paramters describing the cubic are adjusted to provide the desired pressure rise and mass flow for various compressor speeds. The compressor map is made from purely theoretical assumptions, and are not compared with an actual compressor map.

In addition, oil and water is not considered to be present in the gas. Eliminating the effects of wet gas compression and reacting mixtures. The effects of droplets pressent in the gas is to some extent included in the handling of disturbances and in the estimation of degradation.

The system model

In this section, the system models for the compressor in continuous and discrete time will be outlined. The aim is to formulate the dynamic model based on physical laws, and to utilize this to derive an empirical model for the steady-state behavior. Note that no controllers are defined in this section, they are all presented in Section 4.4. The control inputs are however included in some of the equations presented. The model to be investigated is illustrated in Figure 4.2. The system consist of a plenum with compressible gas and two annular ducts where the flow is considered to be incompressible. A throttle valve is placed in the exit duct, and a compressor and a close-coupled valve (CCV) is placed in the inlet duct - all modeled as actuator disks. The compressor and the CCV is placed so close together that no significant mass storage is assumed between the units. With this assumption, the compressor and the CCV work as an equivalent compressor - as the term was introduced by Simon [1993]. The pressure rise across the equivalent compressor is defined as the sum of the pressure rise over the compressor and the pressure drop across the CCV. Thus, the CCV can be used as an actuator for active surge control to perturb the equivalent compressor behavior and suppress oscillations. Using the CCV for surge control is elaborated in Section 4.4.1.

Dynamic behavior

The investigated compression system bears strong resemblance with the one described in Section 3.2.1, as such the same derivation is followed to obtain the dynamic compressor model. Degradation of the compressor is accelerated by the oscillations associated with surge. These oscillations are best monitored as the deviation from the nominal operating point. This, in addition to the design of the regulatory controller, makes the transformed Greitzer model presented by [Simon and Valavani, 1991] a good fit for the dynamic compressor model.

$$\dot{\hat{\phi}} = B[\hat{\Psi}_C(\hat{\phi}) - \hat{\psi} - u] \tag{4.3}$$

$$\dot{\hat{\psi}} = \frac{1}{B}[\hat{\phi} - \hat{\Phi}(\hat{\psi})] \tag{4.4}$$

Here, $\hat{}$ denotes the deviation from the operating points, ϕ_0 and ψ_0, B is the Greitzer parameter, u_{CCV} is the pressure drop across the CCV, $\hat{\Psi}_C$ is the compressor characteristics and $\hat{\Phi}_T$ is the throttle characteristics.

As mentioned in Section 3.1, the operating point of the compressor is at the intersection between the throttle characteristics and the compressor characteristics. In the transformed Greitzer model this is the point $\hat{\Psi}_C = \hat{\Phi}_T$. The compressor characteristic is approximated by the cubicGravdahl and Egeland [1997]:

$$\hat{\Psi}_C(\hat{\phi}) = -k_3\hat{\phi}^3 - k_2\hat{\phi}^2 - k_1\hat{\phi} \tag{4.5}$$

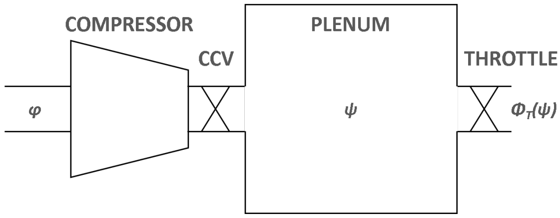

Figure 4.2: Overview of the compression system consisting of a a compressor, a CCV, a plenum, a throttle and ducting between.

where ϕ denotes the non-dimensional mass flow and

$$k_1 = \frac{3H\phi_0}{2W^2}(\frac{\phi_0}{W} - 2) \tag{4.6}$$

$$k_2 = \frac{3H}{2W^2}(\frac{\phi_0}{W} - 1) \tag{4.7}$$

$$k_3 = \frac{H}{2W^3} \tag{4.8}$$

The coefficients, H and W, and the operating point, ψ_0, are greater than 0. The throttle characteristic is given byBacki et al. [2016]:

$$\hat{\Phi_T}(\hat{\psi}) = \gamma \left(\text{sign}(\hat{\psi} + \psi_0)\sqrt{|\hat{\psi} + \psi_0|} - \sqrt{\psi_0} \right) \tag{4.9}$$

Note that $\text{sign}(0) = 0$. The throttle gain, γ, can be calculated from:

$$\gamma = \frac{\phi_0}{\sqrt{\psi_0}} \tag{4.10}$$

The operating point ϕ_0, can ψ_0 be calculated from:

$$\psi_0(\phi_0) = \psi_{0_C} + H\left[1 + \frac{3}{2}(\frac{\phi_0}{W} - 1) - \frac{1}{2}(\frac{\phi_0}{W} - 1)^3\right] \tag{4.11}$$

Steady-state behavior

In the steady-state compressor model, the transient behavior of the compressor is no longer of interest. It is however of interest to utilize the dynamic model for monitoring purposes, and combine this with the steady-state model for prognostic purposes. For this reason the steady-state behavior of the compressor is described by the same dimensionless parameters as the dynamic model, with the operating point defined as (ϕ_0, ψ_0) .

The steady-state behavior of the compressor investigated are specified by a compressor map similar to the one illustrated in Figure 3.1. Each speed line represent the compressor

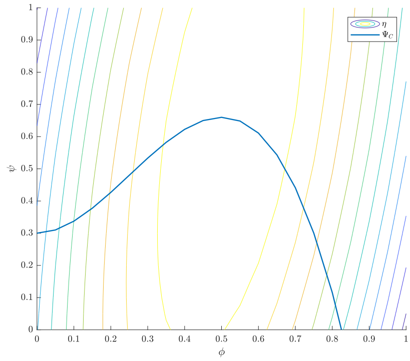

Figure 4.3: Illustration of efficiency islands.

characteristics for a given compressor speed, and is approximated by a cubic on the form:

$$\Psi_C(\phi) = \psi_{c0} + H\left[1 + \frac{3}{2}\left(\frac{\phi}{W} - 1\right) - \frac{1}{2}\left(\frac{\phi}{W} - 1\right)^3\right] \tag{4.12}$$

where W, H and ψ_{c0} are parameters used to fit the speed lines to replicate the compressor behavior.

The efficiency is assumed to be isotropic, and are represented by efficiency islands as ellipsoids on the form:

$$\eta = \eta_{max} - \left(\frac{\phi_0 - c_1}{a}\right)^2 + \left(\frac{\psi_0 - c_2}{b}\right)^2 + \frac{(\phi_0 - c_1)(\psi_0 - c_2)}{a \cdot b} \tag{4.13}$$

where a = -$\frac{0.55}{2} \cdot b$, $b = b(U, \rho)$ and (c_1, c_2) is the center point of the ellipsoid.

The center point is defined as the operating point with highest efficiency. This point is often a little to the right of the peak in the compressor characteristics. In the model the center point is defined to be the operating point where ϕ_0 = 2.1W.

It is assumed that the inlet density and the compressor speed are the parameters that define the steady-state compressor behavior. Thus, the W, H and b values for a given U and ρ is fitted to map the compressor described in Section 4.1. Linear regression is used to fit the various values of the W, H, b and ψ_{c0} parameters to find them as functions of U and ρ:

$$P(\rho, U) = \boldsymbol{\beta}_P \cdot \begin{bmatrix} 1 & U & \rho \end{bmatrix} \tag{4.14}$$

where P indicated the various parameters.

For a given compressor characteristics the operating point is determined by the selection of ϕ_0, as eq.(4.11) yields the corresponding ψ_0 value. Selecting the optimal optimal operating point is done by a trade-off between the efficiency and the distance from the surge point. Remember that this is the peak point in the cubic from eq.(4.12). The trade-off is essentially between low operating costs and amount of damage on the compressor. Running the compressor at the highest possible efficiency are good for the operating costs, since less energy are wasted. However, the closer to the surge point the compressor is operated, the more damage is caused. The distance from the surge point is adjusted by selecting a value for the penalty ω. By solving a optimization problem formulated as:

$$\min_{\phi_0} \qquad J^{SOC} = -\eta(\phi_0) + \lambda\,(2W - \phi_0) \tag{4.15}$$

nonlinear regression is used to find the steady-state correlation for the operating point. (The beta values must be added.)

$$\phi_0^{opt}(\rho, U, \lambda) = 2.5152 - 0.0292 \cdot \rho - 0.0008 \cdot U + 0.0387 \cdot \lambda \tag{4.16}$$

Due to the potential problem of solving nonlinear optimization problems, the above correlation is selected to be the of the lowest degree of nonlinearity with sufficient goodness of fit.

During operation the compressor is subject to disturbances. In the current work, the disturbances are assumed to changing inlet condition - such as temperature, composition, pressure, compressibility and specific-heat ratio. How changes in these variables affect the compressor performance are studied by Lapina [1982]. Based on this study, a quantitative estimation is made to model the disturbances in W, H and ψ_{c0}. All disturbances are assumed to be independent, random and normally distributed.

$$\boldsymbol{D} = \begin{bmatrix} W & H & \psi_{c0} \end{bmatrix}^\top \tag{4.17}$$

with:

$$\mu = \begin{bmatrix} W^{nom} \\ H^{nom} \\ \psi_{c0}^{nom} \end{bmatrix} \qquad \sigma^2 = \mu^\top \cdot \begin{bmatrix} 0.0025 & 0 & 0 \\ 0 & 0.0025 & 0 \\ 0 & 0 & 0.0025 \end{bmatrix} \tag{4.18}$$

Prognostics and health monitoring

The compressor is subject to variations in pressure and flow rate, leading to vibrations, axial thrust displacement and temperature rises which in turn damages important parts of the compressor. Turbo machinery degradation is mainly caused by wear and tear. It is highlighted that the root cause for turbo machinery failure often is excelled degradation or fractions caused by the oscillations associated with surge instabilities. To investigate the effects of un-steady operation and the long-term degradation, the compression system health is monitored within the dynamic model and predicted based on the steady-state operation.

The condition monitoring, diagnostics and prognostics method applied to the compressor are viewed from a steady operating point defined to be optimal. As the compressor operates, disturbances will cause the compressor to drift from the optimal operating point.

This drift could initialize surge, but since a surge controller is employed the oscillations will be damped. These damped oscillations poses a threat to the system health, as they might break the impeller or diffuser blades. It is therefor important to measure these, and based on the amount of oscillations and their amplitude the system health can be assessed as:

$$S(\rho, U, \lambda, \boldsymbol{D}) = \int_0^{\xi_f} \left(p_1|\hat{\phi}| + p_2|\dot{\hat{\phi}}| \right) \rho U A_C d\xi \tag{4.19}$$

Here are p_1 and p_2 weights, and ξ is the dimensionless time variable in the dynamic model.

Due to the properties of the disturbances can the oscillation induced degradation for a given set of compressor settings be calculated as the expected value of $S(\rho, U, \lambda, \boldsymbol{D})$ for a discrete distributed sample of disturbances.

$$S(\rho, U, \lambda) = \mathbb{E}\left[S(\rho, U, \lambda, \boldsymbol{D})\right] = \sum_{\boldsymbol{D}} S(\rho, U, \lambda, \boldsymbol{D}) \cdot P_D(\boldsymbol{D}) \tag{4.20}$$

The probability mass function P_D is found by integrating the probability density function, f_D for an infinitesimal interval. Since all disturbances have a normal distribution, and since they are independent will the oscillation induced degradation have a multivariate normal distribution with the following density function.

$$f_D(\boldsymbol{D}) = \frac{1}{\sqrt{2\pi|\sigma^2|}} e^{-\frac{1}{2}(\boldsymbol{D}-\boldsymbol{\mu})^\top (\sigma^2)^{-1}(\boldsymbol{D}-\boldsymbol{\mu})} \tag{4.21}$$

Degradation of the compressor is caused by several effects. For the current work, degradation is assumed to be caused by the operating load, the oscillations and the compressor speed.

$$\mathcal{D}(\rho, U, \lambda) = k_1\dot{m}(\rho, U, \omega) + k_2 S(\rho, U, \lambda) + k_3 U e^t \tag{4.22}$$

where $\dot{m}$ is the load, and k_1, k_2 and k_3 are weights.

The health state $\mathcal{H}$ is defined to be zero at the start of the time period, meaning that the compressor is of perfect health when operation starts. Over time the compressor health will deteriorate as the degradation increases.

$$\dot{\mathcal{H}} = \mathcal{D} \tag{4.23}$$

The control layers

Regulatory control

The CCV for active surge controlled is presented in more detail by Simon [1993]. To stabilize the compressor beyond the surge line, a linear feedback controller for the CCV is implemented. The proposed linear feedback controller for the CCV is:

$$u_{CCV} = \mu_1\hat{\phi} + \mu_2\hat{\psi} \tag{4.24}$$

where u is the pressure drop across the CCV and μ_1 and μ_2 are parameters. The linear feedback controller is a simplified version of the surge controller presented by Backi et al. [2016] and Backi et al. [2013]. The CCV should be fully opened in the equilibrium point and thus $u \geq 0$. Operating at an initially throttled valve will give the possibility of negative pressure differences, but will lower the performance of the overall compression system [Backi et al., 2016]. Values for the controller parameters are given in Table Table 4.1, the values for the parameters are found from Backi et al. [2013].

Table 4.1: Controller parameter values for simulation of surge controller. Values are from Backi et al. [2013].

Parameter	Value
μ_1	10.07
μ_2	1.83

Supervisory control

In steady-state operation, the compressor will be operating at the point (ϕ_0, ψ_0) calculated to be the optimal by eq.(4.15). Subject to disturbances, the operation may drift from this point. To ensure optimal operation a PI-controller is implemented for self-optimizing control.

The error term is defined as:

$$\Delta c = c_S - c \tag{4.25}$$

where the measurement combination is found from

$$c = \boldsymbol{H} \cdot \boldsymbol{y} \tag{4.26}$$

with

$$\boldsymbol{y} = \begin{bmatrix} \psi_0 & \eta & W & H & \psi_{c0} \end{bmatrix}' \tag{4.27}$$

The set-point c_S is kept constant and calculated by:

$$c_S = \boldsymbol{H} \cdot \boldsymbol{y}^{nom} \tag{4.28}$$

The selection matrix $\boldsymbol{H}$ is found by using the exact local method on the cost function in eq.(4.15).

Health-aware optimization

Economic optimality and satisfaction of operational and reliability constraints are met by adjusting operation for the other layers by this top control layer. A dynamic real-time optimization (DRTO) scheme is devised to calculate the set-points and the optimal penalty weight λ for the SOC layer. At each time step, the dynamic optimization problem is solved:

$$\min_{\lambda,U} \quad J^{DRTO} = -\int_0^{t_f} NPV(\phi_0 \cdot U \cdot \rho)dt = -\int_0^{t_f} \phi_0 \cdot U \cdot (1+i)^{-t}\, dt \tag{4.29}$$

$$\text{subject to} \quad \mathcal{H} < \mathcal{H}_{max} \tag{4.30}$$

$$P_{out}^{min} \leq P_{out} \leq P_{out}^{max} \tag{4.31}$$

$$U^{min} \leq U \leq U^{max} \tag{4.32}$$

$$\lambda^{min} \leq \lambda \leq \lambda^{max} \tag{4.33}$$

$$\tag{4.34}$$

where NPV is the net present value with discount rate i P_{out} is the discharge pressure.

The system is defined by:

$$\frac{d\mathcal{H}}{dt} = \mathcal{D} \tag{4.35}$$

$$\frac{dP_{res}}{dt} = -\dot{m} \tag{4.36}$$

$$m = \phi_0 \cdot \rho A_C U \tag{4.37}$$

$$\Delta P = \psi_0 \cdot \frac{1}{2}\rho U^2 \tag{4.38}$$

$$P_{out} = P_{res} + \Delta P \tag{4.39}$$

$$\rho = \frac{P_{res} M_M}{RT} \tag{4.40}$$

$$\tag{4.41}$$

5

Results and discussion

This section is dedicated to present and discuss the results from applying the control structure to the subsea compressor. First is the regulatory control layer applied to stabilize operation. Second, the self-optimizing controller is tuned and applied to reject disturbances. Finally, the complete control structure will be tested for various operation strategies.

Overview of the control layers

The applied control structure is hierarchical structured into three layers. At the lowest level, a regulatory control layer implement an active surge controller to stabilize the compressor when the load set-point is changed. Moving up the hierarchy, a supervisory control layer implement a self-optimizing controller that adjusts the load set-point to keep the operation optimal in the presence of disturbances. At the top of the hierarchy, a health aware optimization layer finds the optimal operation strategy based on the current and predicted health state of the system so that the specified constraints on the compression and the compressor health is reached. After a control layer is evaluated it will be closed and remain closed for the remainder of the current work.

The investigated compressor are defined by the parameters listed in Table 5.1. All values are collected from Backi et al. [2016].

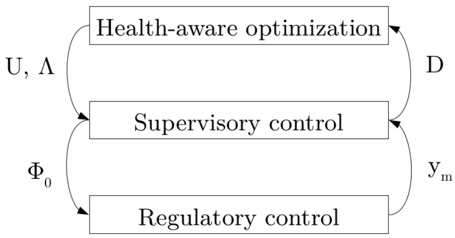

Figure 5.1: Health-aware control structure with information flow

Table 5.1: Values for the compressor specific parameters [Backi et al., 2016].

Parameter	Value	Unit
A_C	0.01	m^2
L_C	3	m
V_P	1.5	m^3
a_s	340	ms^{-1}

Stabilizing the compressor

The purpose of this section is to evaluate the active surge controller presented in Section 4.4.1. For this purpose is the dynamic lumped-parameter model defined by eqs.(4.3)-(4.11) perturbed with a step change in ϕ_0 when the regulatory control loop is open (OL) and when it is closed (CL). The compressor is operating with a compressor speed of 80, corresponding to $W = 0.25$ and $H = 0.18$.

The steady-state compressor characteristics and the throttle characteristics before and after the perturbation visualized in a compressor map are shown in Figure 5.2. It is noted that the steady-state operating points are at the intersections between the cubic compressor characteristics and a throttle characteristics. A perturbation corresponding to $\Delta\phi_0$ = -0.149 yields two different transient responses for the open control loop and the closed loop. Without the regulatory controller, the compressor behavior is contoured as an ellipsoid around the new operating point. With the controller activated, the compressor does not follow the cubic characteristics, but it is able to reach the new operating point.

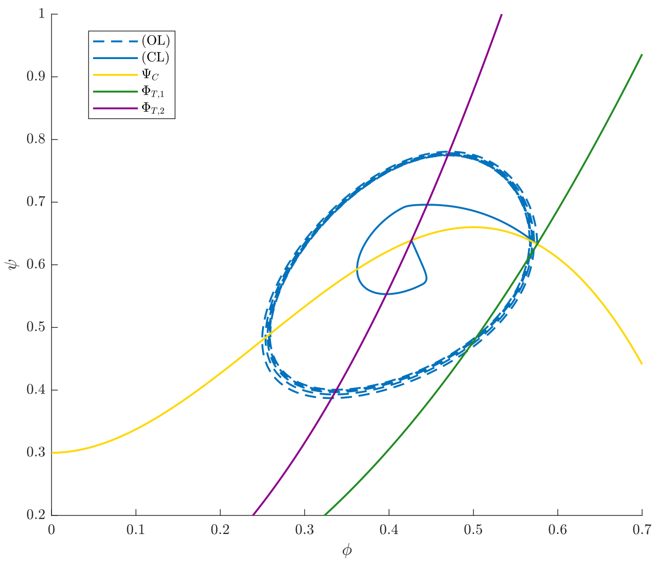

Figure 5.2: The compressor states ϕ and ψ plotted in a compressor map when a step of -0.149 in ϕ_0 perturb the system.

Furthermore, the step responses in ϕ and ψ, as well as the input usage, are plotted against the dimensionless time τ in Figure 5.3. As expected will constant oscillations arise in both load and pressure when the compressor is perturbed into the unstable operating region, if a surge controller is not employed. If the linear feedback controller adjusting the pressure drop across the CCV is employed, will the oscillating response in both load and pressure be so severely damped that the compressor stabilizes at the new operating point well beyond the surge point.

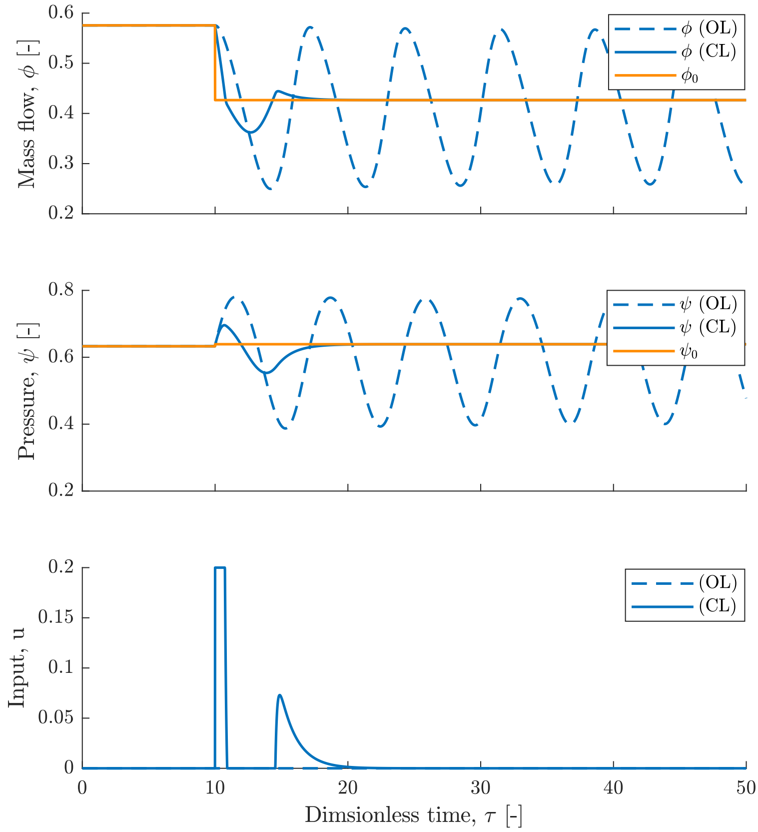

Figure 5.3: The transient responses in load and pressure for open and closed loop, as well as the input usage plotted against dimension less time when a step of -0.149 in ϕ_0 perturb the system.

It can be seen from Figure 5.3 that the input usage is saturated immediately after the step is applied. This raises a concern for the robustness of this controller. If a larger change in the load occurs, will the regulatory controller be able to stabilize the compressor? For the studied operation in the remaining part of the current work this is not an issue, since the applied step in ϕ_0 is much greater than the set-point changes applied by the supervisory control layer.

Optimizing the compressor

The purpose of this section is to evaluate the supervisory controller suggested in Section 4.4.2. First, the supervisory controller is tuned so that operation remains close to optimal in the presence of disturbances. After closing the supervisory control loop, the steady-state compression model is studied to model the expected oscillation induced degradation needed for the diagnostics and prognostics model needed for the health-aware optimization.

Optimal operation is defined according to eq.(4.15) as the point of operation that has the correct trade-off between efficiency and distance from the surge point. With the measurement selection defined by eq.(4.27), the selection matrix $\boldsymbol{H}$ is found by evaluating the loss by the exact local method outlined in Section 2.3.2. For a compressor speed of 890 m/S, a penalty of 1 and inlet density of 36.5 kg/m^3 the selection matrix is found to be:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 1.914 & -0.0075 & -0.9808 & -0.5 \end{bmatrix} \tag{5.1}$$

The corresponding set-point for the controller is:

$$c_S = 2.0087 \tag{5.2}$$

Dynamic open-loop responses in c to a ± 1 % step in ϕ_0 is shown in Figure 5.4. Due to the non-linearity in the efficiency and compressor characteristics, the positive and negative step responses varies. Note that the plotted responses in Figure 5.4 are the absolute value of the response - the response is in fact opposite to the step in ϕ_0. A first-order approximation is used to tune the controller, since this will approximate both responses well. The approximation is selected to be on the form:

$$G = -\frac{0.0175}{2s+1} \tag{5.3}$$

SIMC-rules are used to tune the PI-controller.

$$\begin{aligned}
\tau_c &= 1 \\
K_C &= \frac{1}{k} \cdot \frac{\tau_1}{\tau_c + \theta} = -0.8798 \\
\tau_I &= \min(\tau_1, 4(\tau_c + \theta))2 \\
K_I &= \frac{K_C}{\tau_I} = -0.4399
\end{aligned}$$

To check the performance of the controller the loss are checked for disturbances in W, H and ψ_{c0} all of 1 % of the nominal value. As can be seen from Figure 5.5 are the steady-state loss for the closed loop zero for all disturbances. It is also evident that disturbances in H and $\psi c0$ do not affect the steady-state loss.

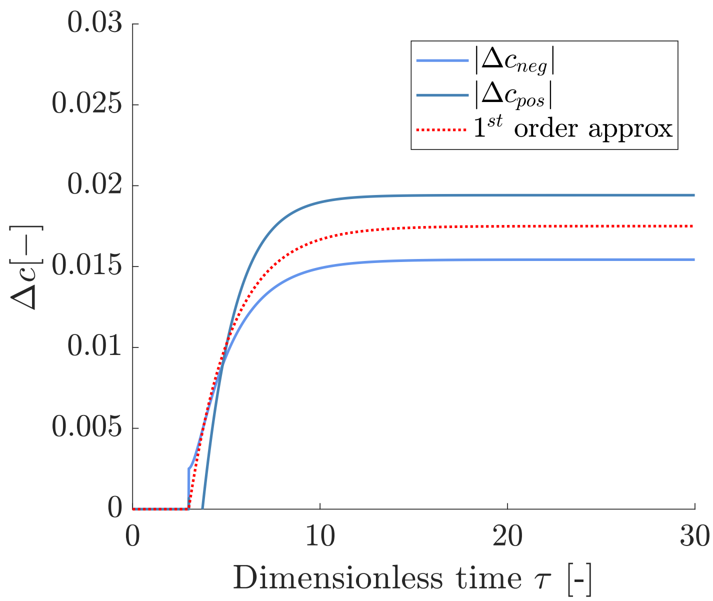

Figure 5.4: Absolute value of pen-loop responses to $\pm$ step changes in the set-point $\phi 0$. The negative (pale blue line) and positive (blue line) are approximated by a 1^{st} order approximation (red line).

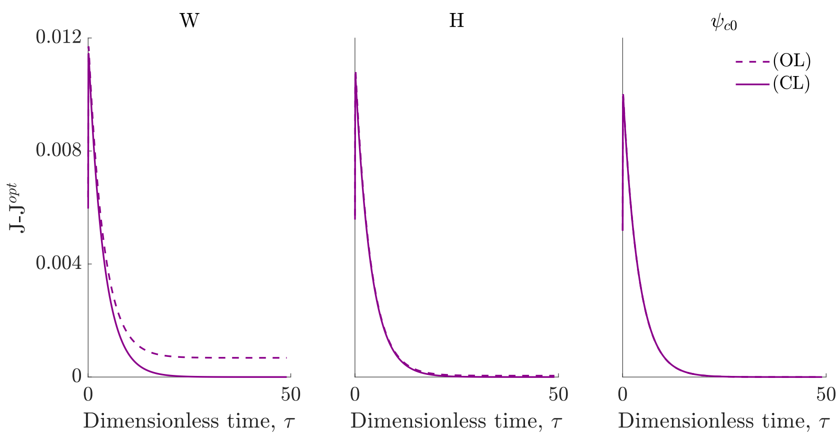

Figure 5.5: Open and closed loop responses in loss for disturbances in H, W and ψ_{c0} with a magnitude of 1 % of their nominal value.

Reassuring system health

The purpose of this section is to evaluate the health-aware optimization layer suggested in Section 4.4.3. The optimization problem is solved in MATLAB using Casadi, IPOPT and the collocation method.

To investigate how the DRTO scheme outlined by eqs.(4.29)-(4.40) is able to reassure the system health for the given time period, three various sets of constraints are implemented. Two of the sets of constraints imposes hard constraints on the penalty, forcing the compressor to operate at certain distances from the surge point. Keeping the penalty high is analog to the so-called surge line mentioned in Section 3.1. Operating along a surge-line is a conservative strategy, for this reason will this optimization strategy define a conservative DRTO. The conservative DRTO do not employ the regulatory and supervisory controllers. Keeping the penalty low is a way of favoring efficiency and the highest possible pressure rise. The health-aware DRTO softens the constraint on the penalty, allowing a larger set of compressor settings. An important difference between the various schemes, is the upper boundary constraint on the health-state. For all schemes except the health-aware, this boundary does not exist. The DRTO schemes are defined in Table 5.2.

Table 5.2: Simulation cases

Strategy	$\mathcal{H}^{max}$	U^{min}	U^{max}	λ^{min}	λ^{max}
Conservative DRTO	∞	550	750	-0.2	0.5
Aggressive DRTO	∞	550	750	3.8	5
Health-aware	183	550	750	-0.8	4

The five year time period the compression is expected to run is divided into 20 decision intervals, where for each interval a set of optimal compressor settings are provided and the operating point and health state is monitored. For each decision interval, the rest of the time horizon is predicted, resulting in a moving time horizon DRTO. At the initiation of each decision interval, the predicted states resulting from the last decision interval is utilized. For two of the decision intervals, the state parameter is not the predicted one, resulting in a changed development of the health state. These changes will not affect the aggressive nor the conservative operational strategies, since these do not consider the system health. The health-aware operation strategy must take these changes in to consideration and the two changes can be viewed as robustness tests.

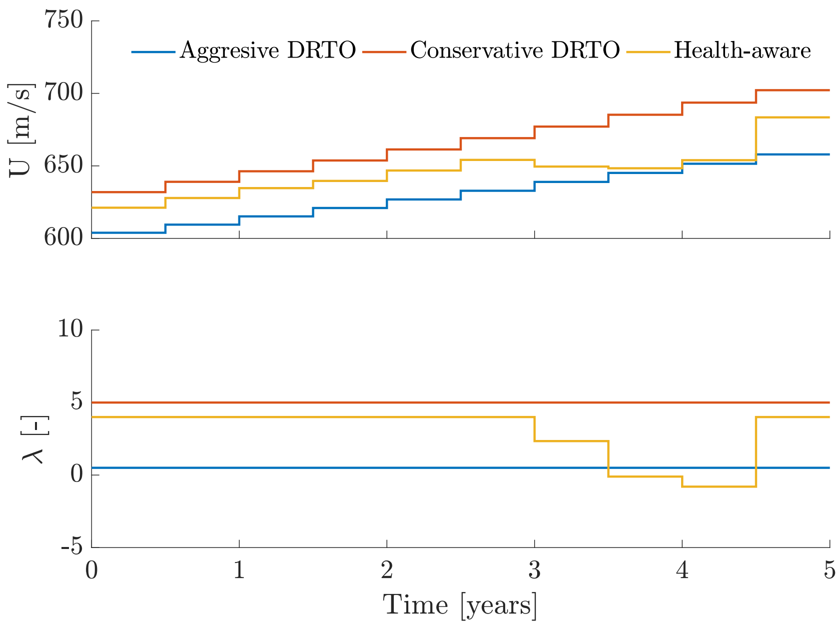

Figure 5.6: The compressor speed and the penalty plotted for the aggressive DRTO, the conservative DRTO and the health-aware DRTO for the total time period.

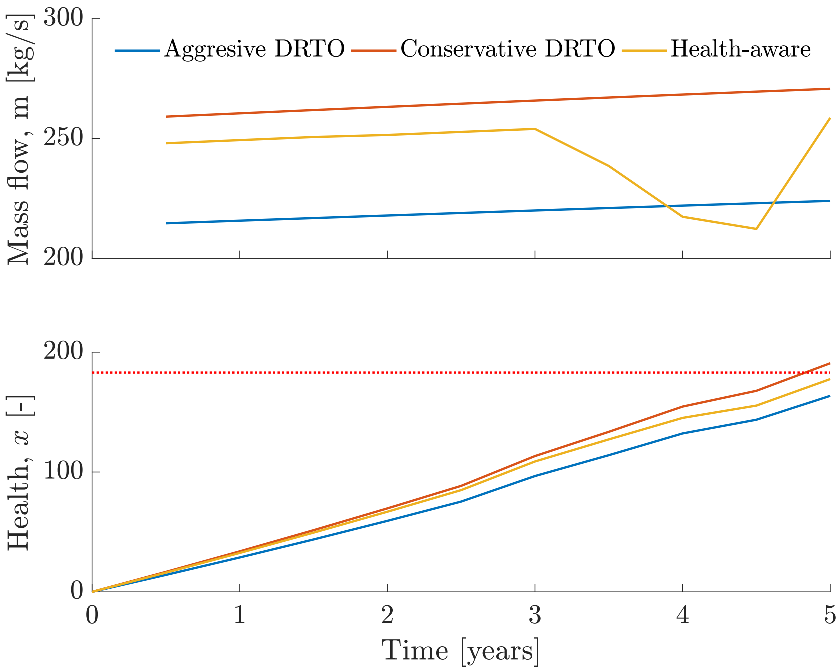

Figure 5.7: The mass flow and health state plotted for the aggressive DRTO, the conservative DRTO and the health-aware DRTO for the total time period.

The conservative, aggressive and health-aware DRTO schemes provide three different operating strategies - as is expected. For the three first years of operation will the penalty be kept constant in all strategies, saturated at the upper boundary. To meet the discharge pressure constraint, the compressor speed U is gradually increased for the following decision intervals. It can be seen from Figure 5.6 that both the aggressive and the conservative operational strategy will follow these trajectories for the entire operation. This is to expect, since the objective of the DRTO is to maximize the net present value of the flow. Thus, with no limitations on the system health, these strategies will have no incentives to operate at lower loads. When the health state is deteriorating faster than predicted, the increase in $\dot{x}$, forces the health-aware DRTO to take action to keep the health state under the threshold value at the end of the prediction period. As a reaction to the worsened health state, the HAC adjust the compressor speed and the penalty to operate at a much lower mass flow. The following reduction in health state deterioration due to the actions of the HAC is visible in Figure 5.7 At $t = 4$ years, the health state deterioration slows down, and the HAC adjust the two following decision interval to speed up production and just meet the health state threshold at the and of the prediction time.

In total production, the conservative DRTO outclasses the other strategies. As a result of the lager total production of gas from the reservoir, the reservoir pressure decreases more for the conservative operation. These effects are plotted in Figure 5.8. Only the HAC have a drop in pressure rise, this is due to the increased deterioration of the system state and the following actions taken by the HAC in the compressor settings.

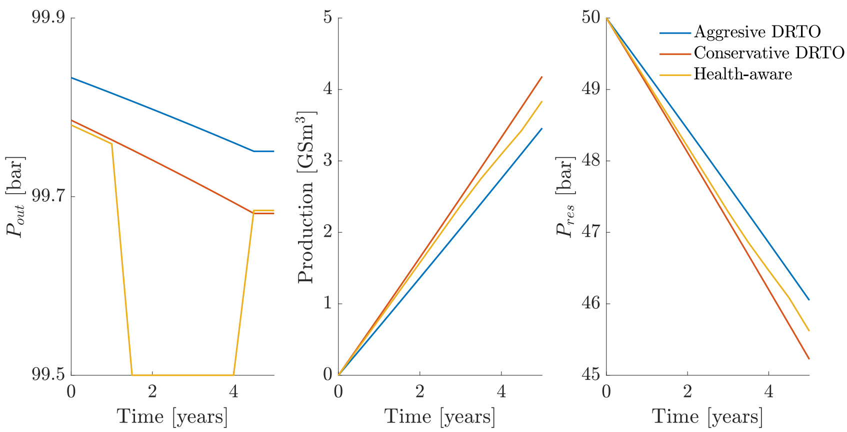

Figure 5.8: Discharge pressure, total production of volumetric gas and the reservoir pressure plotted for the aggressive DRTO, the conservative DRTO and the health-aware DRTO for the total time period.

6

Concluding remarks and further work

A health-aware control structure was applied to a subsea compressor and compared with a typical conservative operation strategy and an aggressive operation strategy. Due to lack of data from a real compressor system, a steady-state model was developed based on a cubic compressor characteristics and estimations of efficiency islands. A dynamic model using Greitzer lumped-parameter dynamic model in the dimensionless form was used to estimate and predict oscillations associated with instabilities in the operation.

As expected, the operational strategy is much dependent on the diagnostics and prognostics of the system health. In the current work, methods for these techniques have only been estimated and no real operating data is used for monitoring. It is evident that the effects of the various operating strategies results in only small differences in the system health, and that huge actions must be taken to meet the health threshold value.

The main conclusion from this work, is that health-aware control has the potential of extending component life time. However, as with other prediction models, accurate models of the system must be developed. Developing and applying monitoring, diagnostics and prognostics models are time consuming, but however crucial for the performance of the health-aware control structure.

Further work

Based on this thesis, it is evident that improvements to the proposed health-aware control structure by:

- implementing more established and proven steady-state compressor models.
- employing diagnostics and prognostics method that consider real life process parameters.

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